

NCERT SOLUTIONS

CLASS-XI MATHS

CHAPTER- 14

MATHEMATICAL REASONING

Exercise 14.1

Q.1: State whether the following sentences are statements or not, and justify your answers.

- (a) A month has 35 days.
- (b) Mathematics is very tough
- (c) Addition of two numbers such as 5 & 7 is larger than 10.
- (d) The resultant of a square of a number is always an even number.
- (e) The arms of a quadrilateral are having equal length.
- (f) Answer the following questions.
- (g) The multiplication result of two numbers such as 8 and (-1) is 8
- (h) The interior angles summed up together results in 180° in a triangle.
- (i) Yesterday was a cloudy day.
- (j) The numbers which are real are always complex numbers.

Sol:

- (a) The maximum number of days in a month is 31. Hence, the statement is incorrect. **Therefore, this isn't a proved statement.**
- (b) Mathematics can be tough for someone and it can be easy for someone too. Hence it is a direct violation to the above question. **Hence, this isn't a statement.**
- (c) The sum of two numbers such as 5 and 7 is 12 which is larger than 10. Hence, the above-mentioned sentence is true. **Therefore, this is a proved statement.**
- (d) Square of all numbers does not give even number as a result. Such as the square of an even number that is 4 is 16 which is an even number, but the square of an odd number such as 9 is 81 which is an odd number. Hence, the above mentioned statement is not fully true. **Therefore, this isn't a proved statement.**
- (e) The above sentence is true only in certain cases. In case of square, all the sides are having equal length whereas in case of a rectangle all the sides are not equal except the opposite sides are equal in length. Hence, above the statement is not true. **Therefore, this isn't a proved statement.**
- (f) Above mentioned sentence is a direct order given to someone in order to answer the upcoming questions. **Therefore, this isn't a proved statement.**
- (g) The multiplication of 8 and -1 is (-8). Therefore, the above mentioned sentence is incorrect. Hence, **this is a proved statement.**
- (h) This sentence is correct because the addition of all interior angles of a triangle is 180° . **Hence, this is a proved statement.**
- (i) The day which is being referred to in the above mentioned sentence is not at all clear. **Therefore, this isn't a proved statement.**
- (j) All real numbers can be written in a format such as $(1) + 0(i)$. Therefore, the given sentence is always correct. **Hence, this is a proved statement.**

Q.2: Give 3 examples of each sentence which are not statements. Give justified reasons for the answers.

Sol:

The three examples of sentences, which are not statements, are as follows.

(i) **Today is a cloudy day.**

The day which is being referred to in the above mentioned sentence is not at all clear. **Therefore, it is not a statement.**

(ii) **Mathematics is very tough**

Mathematics can be tough for someone and it can be easy for someone too. Hence it is a direct violation to the above question. **Therefore, it is not a statement.**

(iii) **Where are you going?**

The above sentence is a question which contains 'you', and it is not at all clear from the above question who is being referred to. **Therefore, it is not a statement.**

Exercise 14.2

Q.1: Write the opposite of the below mentioned statements:

- (a). New Delhi is the capital of India.
- (b). $\sqrt{3+1}$ is a complex number.
- (c). All quadrilaterals are not squares.
- (d). 9 is lesser than 7
- (e). The square of every natural number is a even number

Sol:

- (a) New Delhi is not the capital of India
- (b) $\sqrt{3+1}$ is not a complex number.
- (c) All quadrilaterals are squares.
- (d) 9 is larger than 7
- (e) The square of every natural number is not a even number.

Q.2: State whether the following statements are opposite to each other or not.

- (a). The number 2 is not an even number
The number 2 is not an odd number.
- (b). The number 2 is an even number.
The number 2 is an odd number.

Sol:

- (a). The opposite of the Statement I is "The number 2 is even number ". The above statement is same as Statement II. The reason is if a number is not odd number then it is even number.
- (b). The opposite of Statement I is "The number 2 is not an even number". The above statement is same as Statement II. The reason is if a number is not odd number then it is even number.

Q.3: Find out the component sentences from the below mentioned compound sentences, and determine whether they are true/ false.

- (a). Number 5 is odd or It is a prime number.
- (b). All integers are positive and negative.
- (c). 1000 is divisible by 9 or 10

Sol:

- (a).
 - (i). Number 5 is odd
 - (ii). Number 5 is a prime number.In this case both the statements are true.
- (b).
 - (i). All integers are positive
 - (ii). All integers are negativeIn this case both the statements are false.
- (c).
 - (i). 1000 is divisible by 9
 - (ii). 1000 is divisible by 10In this case only statement (ii) is true.

Q.1:

- (i). Every real number is not complex number and every rational number is a real number.
- (ii). Square of any integer is negative or positive.
- (iii). The sand easily heats up due to the sun but does not cool down easily at night
- (iv). The roots for the equation $x + 10 = 3x^2$ are $x = 3$ and $x = 2$

Sol:

(i) Here 'and' is a connecting word. Here component statements will be:

a: Every real number is not complex.

b: Every rational number are real.

(ii) Here, 'or' is a connecting word. Here component statements will be:

a: Square of any integer is negative.

b: Square of any integer is positive.

(iii) Here 'but' is connecting word. Here component statements will be:

a: The sand heats up easily due to sun.

b: The sand dose not cool down easily at night.

(iv) Here 'and' is the connecting word. Here component statements will be:

a: The roots for the equation $x + 10 = 3x^2$ are $x = 3$ b: The roots for the equation $x + 10 = 3x^2$ are $x = 2$

Q.2: Write negation for the statements after identifying the quantifier for the statements

- (i). There exists one number that is equal to the square of the number
- (ii). For every number that is real 'x', $x < x + 1$
- (iv). There exist one capital for each state of India.

Sol:

(i) The quantifier will be "There exist" and negation for the statement is:

There doesn't exist any number that is equal to the square of the number.

(ii) The quantifier will be "For every" and negation for the statement is:

There exist a number x that is not less than $x + 1$

(iii) The quantifier will be "There exist" and negation for the statement is:

There exist one state that has no capital.

Q.3: Check if the following statements are negation for each other. Justify your answer

- (i). $y + x = x + y$ is true for real numbers x and y
- (ii). There exist real numbers x, y such that $y + x = x + y$

Sol:

The negation for statement (i) will be:

There exist real numbers x, y such that $y + x \neq x + y$, which is not statement (ii).

So the statements are not negation for each other.

Q.4: State if the "Or" in the statements is inclusive or exclusive. Justify the answer

- (i) Moon sets or sun rises
- (ii) You must have ration card or passport for applying a driving license.
- (iii) Integers are negative or positive

Sol:

- (i) "or" is exclusive as it is impossible for moon to set and sun to rise together.
- (ii) "or" is inclusive as one can have both passport and ration card for applying a driving license.
- (iii) "or" is exclusive as integers can't be negative and positive.

Exercise – 14.4

Q.1: Rewrite the statements with 'if & then' in 5 different ways but the sentence should convey the meaning as before.

A natural number is odd implies that its square is odd.

Sol:

- (a). A If the square of the natural number is said to be odd then the number is odd too.
- (b). If the square of a number is not even then the natural number which is squared is also odd.
- (c). It is mandatory that the square of a number to be not even (odd) in order to have that natural number as odd.
- (d). If the natural number is not odd then the square of that number is also not odd that is even.
- (e). In order to have the square of a number (natural number) which is odd, it is sufficient that the number (natural number) is also odd.

Q.2: Rewrite the following sentences as the converse/contrapositive of the followings:

- (a). A quadrilateral is said to be parallelogram if the diagonals bisect each other.
- (b). y is an odd number that is y is divisible by 3
- (c). If 2 lines do not intersect in the same plane, then they are said to be parallel.
- (d). If something is having a low temperature then it implies that is cold
- (e). If you are not able to deduct the reason, then you will not be able to comprehend geometry.

Sol:

(a) **Contrapositive** – If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a parallelogram.

Converse – If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

(b) **Contrapositive** – If y is not divisible by 3 then it is not an odd number.

Converse – If y is divisible by 3, then it is an odd number.

(c) **Contrapositive** – If 2 lines intersect in the same plane, then they are not parallel.

Converse – If 2 lines do not intersect in the same plane, then they are parallel.

(d) **Contrapositive** – If something does not have low temperature, then it implies that it is not cold

Converse – If something is having at a low temperature, then it implies that it is cold.

(e) **Contrapositive** – If you know how to reason deductively, then you can comprehend geometry.

Converse – If you do not know how to reason deductively, then you cannot comprehend geometry.

Q.3: Rewrite the following sentences with "if- then":

- (a). You have visited Qutub Minar implies that you live in Delhi
- (b). You will pass the exam if you study hard.
- (c). In order to get A+ in the class test, you have to do all the problems of that chapter.
- (d). Parallel lines do not intersect each other in the same plane

Sol:

- (a) If you have visited Qutub Minar then you live in Delhi.
- (b) If you study hard then you will pass the exam
- (c) If you want to get A+ in the class test then you have to do all the problems of the book.
- (d) If two lines are parallel then they do not intersect each other in the same plane.

Q.4: Identify the contrapositive/converse from the following sentences:

- (i) If you live in Agra, then you have visited Taj Mahal
- (a) If you have not visited Taj Mahal then you do not live in Agra.
- (b) If you have visited Taj Mahal then you live in Agra.
- (ii) If the diagonals of the quadrilateral bisect each other then that quadrilateral is a parallelogram.
- (a) A quadrilateral is not said to be a parallelogram if the diagonals of a quadrilateral do not bisect each other.
- (b) A quadrilateral is said to be a parallelogram if the diagonals of a quadrilateral bisect each other.

Sol:

- (i).
- (a) Statement I is the contrapositive of the above mentioned statement
- (b) Statement II is the converse of the above mentioned statement
- (ii).
- (a) Statement I is the contrapositive of the above mentioned statement
- (b) Statement II is the converse of the above mentioned statement

Exercise 14.5

Q.1: Prove that p: "If a is real such that $a^3 + 4a = 0$, then a is 0" is true

- (i). by direct method
- (ii). by method of contradiction
- (iii). by method of contra positive

Sol:

p: "if a is real such that $a^3 + 4a = 0$, then a is 0"

Let q: a is real such that $a^3 + 4a = 0$ r, a is 0.

(i). To show that "p" is true, we take that "q" must be true and then prove that "r" is true.

Therefore, assume statement "q" be true.

$$a^3 + 4a = 0 \Rightarrow a(a^2 + 4) = 0$$

$$\Rightarrow a = 0 \text{ or } a^2 + 4 = 0$$

However, since a is real, so it is 0.

Hence, "r" is true.

Thus, the statement is true.

(ii). To show "p" is true using contradiction, we take that "p" isn't true.

Let x be a real number such that $a^3 + 4a = 0$ and let x is not 0.

$$\text{Therefore, } a^3 + 4a = 0 \Rightarrow a(a^2 + 4) = 0 \Rightarrow a = 0 \text{ or } a^2 + 4 = 0 \Rightarrow a^2 = -4$$

However, 'a' is real. Thus, $a = 0$, which is contradiction as we assume that a is not 0.

Hence, the statement "p" is true.

(iii). To prove "p" to be true by using contrapositive method, let r is false and show that q is false.

Here, "r" is false states that its requirement of the negation for statement r.

This obtains the following statement.

$\sim r$. x is not 0.

It is seen $(a^2 + 4)$ cannot be negative so it will be positive.

$a \neq 0$ states that product of a positive number with "a" is not zero.

Let us assume the product of a with $(a^2 + 4)$.

$a(a^2 + 4) \neq 0$

$a^3 + 4a \neq 0$

This proves that "q" isn't true.

Hence, it is proved that $\sim r \Rightarrow \sim q$

Therefore, the statement "p" is true.

Q.2: Prove the statement "For real numbers b and a, $b^2 = a^2$ implies that $b = a$ " isn't true. Give a counter example.

Sol:

Using "if- then" the given statement may be written as follows.

If b and a are two real numbers and $b^2 = a^2$, then $b = a$

Let p: b and a are two real numbers and $b^2 = a^2$

q: $b = a$

To prove: given statement is false. For this we need to prove that p. then $\sim q$. To prove this, we need two numbers b and a with $b^2 = a^2$ such that $b \neq a$ (the numbers must be real numbers)

Let $b = (-1)$ and $a = 1$ $b^2 = (-1)^2 = 1$ and $a^2 = (1)^2 = 1$

Therefore $b^2 = a^2$

However, $b \neq a$

Thus, it is proved that given statement is false.

Q3: By using contrapositive method prove the following statement is true

p: If a is an integer and a^2 is even, then a is even.

Sol:

p: if a is an integer and a^2 is even, then a is even.

Assume q: a is an integer with " a^2 " even. r. a is even

To show: By using contrapositive method p is true, q is false.

r is false [we assume]

let a is odd number

To show that 'q' is false, we have to prove that 'a' isn't an integer or a^2 isn't even

'a' isn't even implies that a^2 is not even

Therefore, 'q' is false. Thus, given statement i.e p is true.