NCERT SOLUTIONS
CLASS-XI MATHS
CHAPTER- 7
PERMUTATIONS AND COMBINATIONS

Exercise- 7.1

Q-1: Form 3- digit numbers by using digits 4, 5, 6, 7 and 8 and assume that

(i) The digits can be repeated.

(ii) Digits can't be repeated in the 3- digit number.

Solution:

(i) There are multiple numbers of ways to form a **3- digit** number by using **5 digits** as there are ways of filling **3 vacant bottles** with **balls** in succession by the given digits.

As per the questions demand, the **repetition** of **digits** is **allowed**. So, the unit places should be filled by any of the **five digits** given in the question.

Similarly,

Tens and hundreds places are filled in by any of the **five digits** given in the question.

Therefore, by the principle of multiplication, by using the given 5 digits i.e., 4, 5, 6, 7, and 8, Total number of 3- digit numbers formed is 5 × 5 × 5 = 125.

(ii) As per the questions demand, the **repetition of the digits are not allowed**. Here, if the units place if filled before other places such as tens and hundreds, then it can be filled by any of the five digits i.e., **4, 5, 6, 7 and 8.**

As the digits can't be repeated and we have **5 digits**, then the total ways of filling the ones place of the **3- digit number is 5.**

Now, remaining numbers that we can use at **tens place is 4**. Now, at tens place, we can use the rest of the 4 digits from the 5 digits given in the question. Similarly, at hundreds place, we can use rest of the 3 digits from the 5- digits given in the question except, that which is already filled at ones place.

By using the **principle** of **multiplication**,

The total number of ways to form a three- digits numbers without repetition of the given digits is 5 × 4 × 3 = 60.

Q-2: Form a 3- digit number which should be even by using 1, 2, 3, 6, 8, and 9. Note that, repetition of digits is allowed.

Solution:

There are multiple numbers of ways to form a **3- digit number** by using **6 digits**, i.e., **1, 2, 4, 6, 8 and 9,** as the way we fill **3 vacant bottles** with **balls** in succession by the given balls.

The number should be an **even number**. So, the unit places should be filled by either 2 or 6 or 8, i.e., we can fill the **ones place** in **3 ways.**

As per the question demand, the **repetition of digits is allowed**. So, the **tens place** should be filled by either of the given **6 digits, i.e., 1, 2, 4, 6**, 8 **and 9** and the **hundreds place** should also be filled by either of the **6 digits i.e., 1, 2, 4, 6**, 8 **and 9** in **6** different ways.

Therefore, by the principle of multiplication, by using the given 6 digits i.e., i.e., 1, 2, 4, 6, 8 and 9, Total of 3- digit even numbers formed is 3 × 6 × 6 = 108.

Q-3: Find the total number of 4- letter code formed by using the 10 English alphabets, i.e., a, b, c, d, e, f, g, h, i and j. Assume that the repetition of the letters of English alphabet is not allowed.

Solution:

There are multiple numbers of ways to form a **4- letter code** ;by using **10 letters** from the English alphabet, i.e., **a, b, c, d, e, f, g, h, i and j,** as the way we fill 4 vacant bottles with balls in succession by the given balls.

Repetition of the letters from English alphabet is **not allowed.**

So,

If we start filling up the letters from ones place, then we can fill it in **10 different ways** as we have **10 letters** from the **English alphabets**.

Now, we can fill tens place in **9 different ways** from the remaining **9 letters from the 9 English alphabets**, and also on hundreds place we can fill it by **8 different ways from the remaining 8 letters** from the English alphabets, and at hundreds place, we can fill it in **7 different ways from the remaining 7 letters** from the English alphabets as the **repetition** is **restricted**.

Hence, by the **principle of multiplication**, by using the given **10 letters** from the English alphabets, i.e., a, b, c, d, e, f, g, h, i and j, total of 4- letter code formed is 10 × 9 × 8 × 7 = **5040.**

Therefore, 5040 four- letter code is formed by using the given 10 letters i.e., a, b, c, d, e, f, g, h, i and j from the English alphabets, and there is no repetition in the code hence formed.

Q-4: Find the total number of 5- digit telephone numbers formed by using the numbers from 0 to 9. Note that, each of the telephone number will start with 36 and the repetition of the numbers is not allowed.

Solution:

In the question, it is given that the **telephone number** is of 5 **digits** and should start with **36.**

So, there will be several telephone numbers as we have to get **5- digit number** where **2 digit** is fixed that is the first two numbers will be **3 and 6**, respectively, i.e., 3, 6, _, _, _ by using the digits from **0 to 9**, **except** the **digits 3 and 6**.

If we start filling up the numbers to form a telephone number, from **ones place** then we can fill it in **8 different ways** as we have **8 digits from the 0 – 9**.

Now, we can fill **tens place** in **7 different ways** from the remaining **7 digits from the 0 to 9**, and also on hundreds place we can fill it in **6 different ways** from the remaining **6 digits** from the **0 to 9**, as the **repetition** is **restricted**.

Hence, by the principle of multiplication, the total number of ways to form 5-digit telephone number is 8 × 7 × 6 = 336.

Q-5: Consider a scenario when a person tosses a coin. He tosses the coin for two times and every time the outcome of the toss is recorded by one of his friend. Find the total number of possible outcomes of the coin his friend will record.

Solution:

A **person tossed** a **coin** for **two times** and recorded the outcome every time.

Whenever the **coin** will be **tossed**, the number of **outcome of the coin is 2** which are usually called as a head or a tail or we can say that, in each throw, the different ways to show a **different face is 2.**

Therefore, by the principle of multiplication, the total number of the outcomes possible by the throw can be 2 × 2 = 4.

Q-6: Consider 5 flags which are of different colors. Find the total possibility of getting different signals. Note that, each of the signals requires the use of any two flags at the same time, one after the other.

Solution:

As per the scenario given in the question, **each** of the **signals** uses **2 flags** at a time.

There are so many possibilities of the combination as the number of ways of filling the **2 vacant boxes** in succession of the given **5 flags** which are of **different color**.

Now, let us start with the upper portion of the signal be filled with **5 different ways with any of the 5 flags**. After picking up one, the lower portion of the signal will be followed by the upper one, which must be filled up with **4 different signals** by any one of the **4 different colored flags left**.

Therefore, by the principle of multiplication, the total number of the signals which will be generated by 5 different colors of flag is 5 × 4 = 20.

EXERCISE- 7.2

Q-1: Evaluate:

(i). 8! **(ii). 4! – 3!**

Solution:

(i). 8! = 1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 **= 40320**

(ii). 4! = 1 × 2 × 3 × 4 **= 24**

And, 3! = 1 × 2 × 3 **= 6**

Therefore, 4! – 3! = 24 – 6 = 18

Q-2: Is 3! + 4! = 7! ?

Solution:

Now, 3! = 1 × 2 × 3 **= 6**

4! = 1 × 2 × 3 × 4 **= 24**

And, 7! = 1 × 2 × 3 × 4 × 5 × 6 × 7 **= 5040**

Therefore, 4! – 3! ≠ 7!

Q-3: Compute:

$$\frac{9!}{5! \times 3!}$$

Solution:

$$\frac{9!}{5! \times 3!} = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9}{1 \times 2 \times 3 \times 4 \times 5 \times 1 \times 2 \times 3}$$

Therefore, $\frac{9!}{5! \times 3!}$ **= 392**

Q-4. If $\frac{1}{5!} + \frac{1}{6!} = \frac{a}{7!}$**, find the value of a.**

Solution:

$$\frac{1}{5!} + \frac{1}{6!} = \frac{a}{7!}$$

$$\Rightarrow \frac{1}{5!} + \frac{1}{6 \times 5!} = \frac{a}{7 \times 6 \times 5!}$$

$$\Rightarrow \frac{1}{5!} + \left[1 + \frac{1}{6}\right] = \frac{a}{7 \times 6 \times 5!}$$

$$\Rightarrow 1 + \frac{1}{6} = \frac{a}{7 \times 6}$$

$$\Rightarrow \frac{7}{6} = \frac{a}{7 \times 6}$$

$$\Rightarrow a = \frac{7 \times 7 \times 6}{6}$$

Therefore, a = 49

Q-5: Evaluate: $\frac{a!}{[illegible]}$**, when**

$(a-r)!$

(i). a = 5, r = 1 (ii). a = 8, r = 4

Solution:

(i) a = 5, r = 1: [Given]

$\frac{a!}{(a-r)!} = \frac{5!}{(5-1)!} = \frac{5!}{4!} = \frac{5 \times 4!}{4!}$

Therefore, $\frac{a!}{(a-r)!}$ **= 5**

(ii) $\frac{a!}{(a-r)!} = \frac{8!}{(8-4)!} = \frac{8!}{4!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!} = 8 \times 7 \times 6 \times 5 = 15120$

Therefore, $\frac{a!}{(a-r)!}$ **= 15120**

EXERCISE – 7.3

Q-1: If there is no repetition of numbers, then find the total of 3- digit numbers formed by using digits from 2 to 9.

Solution:

Digits in **between 2 to 9 are 8.**

We have to construct 3- digit numbers by using the digits from 2 to 9.

It is given that; the **repetition of digit is restricted**.

So, from the **permutation** point, there are **permutations** of 8 digits (different) among which we need to take 3 at a time.

Hence, **the total of 3- digit number formed by using digits from 2 to 9** =
$^8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!}$

$= \frac{8 \times 7 \times 6 \times 5!}{5!}$ **= 336**

Therefore, total of 336 3-digit numbers can be formed by using digits from 2 to 9 without any repetition.

Q-2: Find the total number of 4- digits numbers where there is no repetition of digits.

Solution:

We have to construct 4- digit numbers from 0 to 9.

Repetition of digit is restricted.

We know that to have a proper number the thousands place must be filled with **9 digits from 1 to 9**, because **0** will be considered as **negligible** at **thousands**

place.

Therefore, the **total number of ways** through which the **thousands place** should be filled **is 9.**

Now, the hundreds place must be filled with remaining **9 digits between 0 to 9**, except that number which is inserted at thousands place, as the **repetition is restricted**. Similarly, **tens and ones place** can be filled by the **remaining 9 digits.**

Hence, for hundreds, tens and ones, there will be 3- digit number and therefore, the **permutations** of 9 different digits will be taken **3 at the same time**.

Now,

The total number of 3- digit number:

= ${}^9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!}$

= $\frac{9 \times 8 \times 7 \times 6!}{6!}$ = 9 × 8 × 7 = 504

Therefore, by the principle of the multiplication, the total number of the 4 – digit number is 504 × 9 = 4536

Q-3: Find the total number of 3- digit number which is even and is formed by using the digits 2, 3, 4, 5, 6, 7 and 8. Note that repetition of digit is restricted.

Solution:

We need to form even numbers of **3- digits** which can be formed by using the digits such as **2, 3, 4, 5, 6, 7 and 8. Repetition of the digit is restricted.**

To make the **3- digit number even**, the **unit place** must be filled with an **even number** that is **2, 4, 6, and 8** among the given number.

As it is given in the question that the **repetition of the numbers** is restricted so, the **unit place** must be already occupied with an even number, and the **hundreds** and **tens place** is to be filled with the **remaining 6 digits** given in the question.

Hence, to fill up the **hundred** and **tens place** we need to pick up digits except that which is being filled on **ones place**. So, we need to pick with **permutation of 6 different digits** among which **2 digits** are **taken** at a time.

Hence, the **number of ways through which the hundreds and tens place is filled:**

= ${}^6P_2 = \frac{6!}{(6-2)!} = \frac{9!}{4!}$

= $\frac{6 \times 5 \times 4!}{4!}$ **= 30**

Therefore, by the principle of the multiplication, the number of the 3- digit numbers required is 4 × 30 = 120.

Q-4: How many 4- digit numbers will be formed by using the digits 2, 3, 4, 5 and 6. Note that, repetition of digits is not allowed. Find among those numbers, how many numbers will be even number?

Solution:

We need to construct 4 – digit numbers using the digits 2, 3, 4, 5 and 6.

We have **5 digits** among which we need to form **4- digit** number as there are **permutations of 5 different digits taken 4 at a time.**

Hence, the required number of the 4- digit numbers:

$$= {}^5P_4 = \frac{5!}{(5-4)!} = \frac{5!}{1!}$$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{1} = 120$$

Among these **120 numbers** of **4 – digit** which is formed by using **2, 3, 4, 5 and 6**, even numbers must end with an even number which is **2, 4, or 6**.

There are chances that there are **3! Number** of ways through which the ones place is filled with **digits**, i.e., it can be filled by **6 number of ways.**

As, in the question, it is mentioned that repetition of **digits are restricted** and the **units place** is already occupied with an **even number**, so the remaining place is to be filled by the **remaining 4 digits** which is not inserted at any place.

Hence, the number of ways by which the **thousands**, **hundreds** and **tens place** will be filled is the **permutation** of **4 different digits** which is taken **3 at a time**.

Therefore, the number of ways to fill the remaining places:

$$= {}^4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!}$$

$$= \frac{4 \times 3 \times 2 \times 1}{1} = 24$$

Hence, by the principle of multiplication, the required number which is an even number is 24 × 3 = 72.

Q-5: Consider a committee with 9 persons, in how many ways can we choose a chairman and a vice chairman? Note that, a person can hold only one position at a time.

Solution:

There is a **committee** with **9 persons** among which, **a vice- chairman and a chairman** is chosen in such a way that **one position can be held by only one person**.

So,

To choose a **chairman** and **a vice- chairman**, there are number of ways with **permutation** of **8 different persons taken 2 at a time.**

Therefore, the number of ways required:

$$= {}^9P_2 = \frac{9!}{(9-2)!} = \frac{9!}{7!}$$

$$= \frac{9 \times 8 \times 7!}{7!} = 72$$

Q-6: Find n, if ${}^{n-1}P_3 : {}^{n}P_4$ = 1 : 9.

Solution:

$${}^{n-1}P_3 : {}^{n}P_4 = 1:9$$

$$\Rightarrow \frac{{}^{n-1}P_3}{{}^{n}P_4} = \frac{1}{9}$$

$$\Rightarrow \frac{\left[\frac{(n-1)!}{(n-1-3)!}\right]}{\left[\frac{n!}{(n-4)!}\right]} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{n \times (n-1)!} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9}$$

Therefore, n = 9

Q-7: Find the value of a, if:

(i). ${}^{5}P_a = 2\left[{}^{6}P_{a-1}\right]$ (ii). ${}^{5}P_a = {}^{6}P_{a-1}$

Solution:

(i). ${}^{5}P_a = 2\left[{}^{6}P_{a-1}\right]$

$$\Rightarrow \frac{5!}{(5-a)!} = 2 \times \frac{6!}{(6-a+1)!}$$

$$\Rightarrow \frac{5!}{(5-a)!} = \frac{2 \times 6!}{(7-a)!}$$

$$\Rightarrow \frac{5!}{(5-a)!} = \frac{2 \times 6 \times 5!}{(7-a)(6-a)(5-a)!}$$

$$\Rightarrow 1 = \frac{2 \times 6}{(7-a)(6-a)}$$

So, (7 – a) (6 – a) = 12

$$\Rightarrow 42 - 6a - 7a + a^2 = 12$$

$$\Rightarrow a^2 - 13a + 42 = 12$$

$$\Rightarrow a^2 - 13a + 30 = 0$$

$$\Rightarrow a^2 - 3a - 10a + 30 = 0$$

$$\Rightarrow a(a-3) - 10(a-3) = 0$$

$$\Rightarrow (a-3)(a-10) = 0$$

So, a = 3 or a = 10

It is known that, ${}^{n}P_r = \frac{n!}{(n-r)!}$, $where\ 0 \le a \le n$

$$\Rightarrow 0 \le a \le 5$$

Therefore, a ≠ 10

Hence, r = 3

(ii). ${}^5P_a = {}^6P_{a-1}$

$\Rightarrow \frac{5!}{(5-a)!} = \frac{6!}{(6-a+1)!}$

$\Rightarrow \frac{5!}{(5-a)!} = \frac{6!}{(7-a)!}$

$\Rightarrow \frac{5!}{(5-a)!} = \frac{6 \times 5!}{(7-a)(6-a)(5-a)!}$

$\Rightarrow 1 = \frac{6}{(7-a)(6-a)}$

So, (7 – a) (6 – a) = 6

$\Rightarrow 42 - 6a - 7a + a^2 = 6$

$\Rightarrow a^2 - 13a + 42 = 6$

$\Rightarrow a^2 - 13a + 36 = 0$

$\Rightarrow a^2 - 4a - 9a + 36 = 0$

$\Rightarrow a(a-4) - 9(a-4) = 0$

$\Rightarrow (a-4)(a-9) = 0$

So, a = 4 or a = 9

It is known that, ${}^nP_r = \frac{n!}{(n-r)!}$, $where\ 0 \le a \le n$

$\Rightarrow 0 \le a \le 5$

Therefore, a ≠ 9

Hence, a = 4

Q-8: Get all the number of ways, with or without the meaning, which might be formed by using the letters from the word EQUATION. Note that, repetition of the word is restricted.

Solution:

In the word **EQUATION**, we have **8 letters** from the **English alphabets**.

Hence, to find the number of words formed by using all those 8 letters of the word **EQUATION**, without repetition of the alphabets, we have:

Permutation of **8 different letters taken 8 at a time**, which is ${}^8P_8$ **= 8!**

Hence, the number of words required which can be formed = 8! = 40320.

Q-9: Find the number of words, with or without the meaning, which might be formed by using the letters from the word MONDAY. Note that, repetition of the word is restricted, if

(i) At a time. 3 letters can be used.

(ii) **All the letters can be used at a time.**

(iii) First letter is a vowel and all the letters are being used.

Solution:

In the word **MONDAY**, we have **6 letters** from the **English alphabets**.

(i) As per the question demand, we need to use 3 letters at a time.

So, the number of words formed by the letters of the word MONDAY, without the repetition of the letters, is

The permutation of 6 different objects which are taken 3 at a time is given by, ${}^6P_3$

Hence, the number of words required which can be formed by using the word MONDAY using only 3 letters at a time is given by,

${}^6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \Rightarrow \frac{6 \times 5 \times 4 \times 3!}{3!} = 6 \times 5 \times 4 =$ **120 words**

(ii) In this case, at a time, we can use all the letters of the given word **MONDAY**.

So, to find the number of words formed, we have:

Permutation of 6 different letters taken any of the **6 at a time**, that is: ${}^6P_6$.

Hence, **the number of words required** which can be formed by using all the letters of the word **MONDAY** at a time = **6! = 6 × 5 × 4 × 3 × 2 × 1 = 720.**

(iii) In the given words **MONDAY**, we have two vowels which is **O and A.**

As per the given condition, vowels must be filled at the start of the word. Also, the **repetition of letter is restricted.**

So, to this can be done only in **2! Number of ways**.

We have **6 letters** and the **repetition is restricted**, also the rightmost place will be filled by a **vowel** from the **word MONDAY**, so the remaining **5 places** will be filled by the remaining **5 letters**. And, this can be done in **5! Number of ways.**

Hence, in this case, the number of words required and being formed is 5! × 2! = 5 × 4 × 3 × 2 × 1 × 2 × 1 = 240.

Q-10: Find the number of distinct permutations such that 4 I's does not come together in the word MISSISSIPPI.

Solution:

In the word **MISSISSIPPI, M appears for 1 time, I appears for 4 times, P appears for 4 times and S appears for 4 times.**

So, the required number of the distinct **permutations** of the letters in the word **MISSISSIPPI** is given by:

$\frac{11!}{4! \times 4! \times 2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 4 \times 3 \times 2 \times 1 \times 2 \times 1} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1 \times 2 \times 1}$ **= 34650.**

As there are **4 I's** in this given number which is **MISSISSIPPI**, whenever they will occur together, they will be treated as a single object which is **IIII.** Since, all the **4 I's** are considered as **1** and there are **7 letters** left in the given word **MISSISSIPPI**, so we have total of **8 words** in account.

Among these **8 words S** appears for **4 times** and **P** appears for **4 times**, and **M** appears just **once**.

So, this can be arranged in $\frac{8!}{4! \times 2!}$ ways, i.ie., 8 × 7 × 3 × 5 = **840 ways**

So, the **number** of **arrangements** where **all I's** can occur together **= 840**

Hence, the total number of the distinct permutations of the letters in the given word MISSISSIPPI, where all the 4 I's won't come together = 34650 – 840 = 33810.

Q-11: Arrange the letters of the word PERMUTATIONS and observe the number of ways of the permutation if,

(i). the new word starts with P and ends with S.

(ii). when the vowels are taken all together.

(iii). there will always be 4 letters between P and S.

Solution:

In the given word **PERMUTATIONS**, all the other letters occurred for once except **T** which occurred for **twice**.

(i) As we need to fix **P** and **S** at the extreme ends i.e., **P** at the **left side** of the **word** and **S** on the **right side** of the **word**, then we have **10** more letters to fix.

Therefore, number of arrangements required in this case = $\frac{10!}{2!}$ **= 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 = 1814400**

(ii) In the given word **PERMUTATIONS**, we have all the **5 vowels**, i.e., **a, e, i, o, u** and each of the vowel is appearing only for once in the given word.

As in this case, they have to occur together, so these vowels are considered as a single object. Since, we have **12 letters** in the given word and as all the **vowels** are counted as one, so we have more **7 letters**. So, **8 objects** are there in the account.

Now,

All these letters can be arranged in $\frac{8!}{2!}$ **ways** where there is **2 T's**.

As there are 5 different vowels, then corresponding to these arrangements, **vowels** will be arranged in **5! Ways.**

Hence, by the principle of multiplication, number of arrangements required for this case = $\frac{8!}{2!} \times 5!$ **= 2419200.**

(iii) We have to arrange the letters of the word **PERMUTATIONS** in such a way that always there will be **4 letters in between P and S.**

So, it is clear that the **position of P and S** is fixed and **4** others letters can be inserted between **P and S**. There are **2 T's** in the remaining **10 letters** which can be arranged in $\frac{10!}{2!}$ ways.

Also, **4 letters** can be placed in between P and S in **2 × 7 = 14 ways.**

Hence, by the principle of multiplication, number of arrangements required in this case = $\frac{10!}{2!} \times 14$ **= 25401600.**

EXERCISE – 7.4

Q-1: If $^nC_8 = ^n C_3$, find the value of $^nC_3$

Solution:

We know that, if $^nC_x = \ ^nC_y$

So, x = y or n = x + y

Hence, $^nC_8 = \ ^nC_3$

n = 8 + 3 = 11

Therefore, $^nC_2 = \ ^{11}C_2 = \frac{11!}{2!(11-2)!} = \frac{11 \times 10 \times 9!}{2 \times 1 \times 9!} = \frac{11 \times 10}{2}$

Therefore, $^nC_8 = \ ^nC_3$ = 55

Q-2: Find the value of 'a' if:

(i) $^{2a}C_2 : \ ^aC_2$ = 12 : 1

(ii) $^{2a}C_2 : \ ^aC_2$ = 11 : 1

Solution:

(i). $\frac{^{2a}C_2}{^aC_2} = \frac{12}{1}$

$$\Rightarrow \frac{(2a)!}{2!(2a-2)!} \times \frac{2!(2a-2)!}{a!} = \frac{12}{1}$$

$$\Rightarrow \frac{(2a)(2a-1)(2a-2)!}{(2a-2)!} \times \frac{(a-2)!}{a(a-1)(a-2)!} = 12$$

$$\Rightarrow \frac{(2)(2a-1)}{(a-1)(a-2)} = 12$$

$$\Rightarrow \frac{4(a-1)(2a-1)}{(a-1)(a-2)} = 12$$

$$\Rightarrow \frac{(2a-1)}{(a-2)} = 3$$

So, 2a – 1 = 3(a – 2)

2a – 1 = 3a – 6

3a – 2a = -1 + 6

Therefore, the value of a = 5

(ii). $\frac{^{2a}C_2}{^{a}C_2} = \frac{11}{1}$

$$\Rightarrow \frac{(2a)!}{2!(2a-2)!} \times \frac{2!(2a-2)!}{a!} = \frac{11}{1}$$

$$\Rightarrow \frac{(2a)(2a-1)(2a-2)!}{(2a-2)!} \times \frac{(a-2)!}{a(a-1)(a-2)!} = 11$$

$$\Rightarrow \frac{(2)(2a-1)}{(a-1)(a-2)} = 11$$

$$\Rightarrow \frac{4(a-1)(2a-1)}{(a-1)(a-2)} = 11$$

$$\Rightarrow 4(2a-1) = 11(a-2)$$

So, 8a – 4 = 11(a – 2)

8a – 4 = 11a – 22

11a – 8a = -4 + 22

3a = 18

Therefore, the value of a = 6

Q-3: Find total number of chords which can be drawn through 21 points on the circle.

Solution:

2 points are required to draw one chord on a circle.

Now, in order to obtain the number of chords which can be drawn through the given **21 points** on the circle, we need to count **total number of the combinations**.

Hence, there can be several numbers of chords as there are combinations of **21 points** which can be taken **2 at a time.**

Therefore, number of chords required = $^{21}C_2 = \frac{21!}{2!(21-2)!} = \frac{21!}{2!\ 19!} = \frac{21 \times 20}{2}$

= 210

Q-4: Find the number of ways in which a team of 2 boys and 2 girls can be selected from the group of 5 boys and 4 girls.

Solution:

We have to select **2 boys and 2 girls** from a group of **5 boys and 4 girls** to form a team.

Now,

2 boys should be selected among **5 boys** group in $^5C_2$ **ways.**

2 boys should be selected among 5 boys group in $^5C_2$ ways.

Also, **2 girls** should be selected among a group of **4 girls** in $^4C_2$ **ways.**

Hence, by the **principle of multiplication**, the number of ways by which a **team of 2 boys and 2 girls** can be formed:

$= {}^5C_2 \times {}^4C_2 = \frac{5!}{2!\,3!} \times \frac{4!}{2!\,2!} = \frac{5 \times 4 \times 3!}{2 \times 1 \times 3!} \times \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} = 5 \times 2 \times 2 \times 3$ **= 60 ways**

Q-5: In how many ways will 9 balls be selected from 7 red balls, 6 blue balls and 5 white balls if every time 3 balls of different colors be selected?

Solution:

We have total of **6 blue balls, 5 white balls and 7 red balls.**

We have to select **9 balls** from these **18 balls** in such a ways that every time **3 different colored balls** can be selected.

Here,

We can select **3 balls** from **7 red balls** in $^7C_3$ number of ways.

We can select **3 balls** from **6 blue balls** in $^6C_3$ number of ways.

We can select **3 balls** from **5 white balls** in $^5C_3$ number of ways.

Therefore, by **the principle of multiplication**, number of ways required to select **9 balls**:

$= {}^7C_3 \times {}^6C_3 \times {}^5C_3 = \frac{7!}{3!\,4!} \times \frac{6!}{3!\,3!} \times \frac{5!}{3!\,2!} = \frac{7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 1} \times \frac{6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2 \times 1} \times \frac{5 \times 4 \times 3!}{3! \times 2 \times 1}$

= 7000 ways

Q-6: Find out the number of combinations of 5 cards from a deck of 52 cards. Assume that there is at least one ace in each of the combinations.

Solution:

We know that, we have **52 cards** in a deck where there are **4 aces.**

We have to make a combination of **5 cards** in which there will be one ace for sure.

As we have **4 aces** in the deck so, aces can be selected in $^4C_1$ number of ways and out of 48 cards in the deck, 4 cards can be selected in $^{48}C_4$ **number of ways**.

Therefore, by the **principle of the multiplication**, number of the combination of **5 cards** required:

$= {}^{48}C_4 \times {}^4C_1 = \frac{48!}{4!\,44!} \times \frac{4!}{1!\,3!} = \frac{48 \times 47 \times 46 \times 45 \times 44!}{4 \times 3 \times 2 \times 1 \times 44!} \times \frac{4 \times 3!}{3!}$

$= \frac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2 \times 1} \times 4$

= 778320 number of combinations.

Q-7: Find the number of ways by which one can select a cricket team of 11 players from a bunch of 17 players among which only 5 players can bowl if every cricket team of 11 players must have exactly 4 bowlers.

Solution:

Among 17 players, 5 players can bowl.

Now, we need to select a cricket team of **11 players** where there are exactly **4 players** who **can bowl**.

So, Among 5, **4 bowlers** can be selected in $^5C_4$ **number of ways** and the remaining **7 players** for the team of **11 players** will be selected from the remaining **12 players** in $^{12}C_7$ **number of ways.**

Therefore, by the **principle of multiplication**, number of ways required to select a cricket team:

$= {}^5C_4 \times {}^{12}C_7 = \frac{5!}{4!\ 1!} \times \frac{12!}{7!\ 5!}$

$= \frac{5 \times 4!}{4!} \times \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7!}{7! \times 5 \times 4 \times 3 \times 2 \times 1}$

$= \frac{5 \times 12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1}$

= 3960 number of ways.

Q-8: There are 6 black and 5 red balls. Find the number of ways through which 3 black and 3 red balls will be selected.

Solution:

We have a bag which contains **6 black** and **5 red balls**.

3 black balls will be selected from **6 black balls** in $^6C_3$ **number of ways** and **3 red balls** will be selected out of **5 red balls** in $^5C_3$ **number of ways.**

Therefore, **by the principle of multiplication**, required number of ways to select **3 black and 3 red balls**:

$= {}^6C_3 \times {}^5C_3 = \frac{6!}{3!\ 3!} \times \frac{5!}{3!\ 2!} = \frac{6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2 \times 1} \times \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} = 5 \times 4 \times 5 \times 2$

= 200 number of ways.

Q-9: Find the number of ways by which a student can choose a program of 5 courses, if there are options of 10 courses and for every student 2 courses are made compulsory.

Solution:

10 courses are available out of which **2 courses** are made compulsory for each student.

Hence, each student have to choose **3 courses** from the remaining courses, i.e., **8 other remaining courses**, which can be selected in $^8C_3$ **number of ways.**

Therefore, the total number of ways in which **program** can be chosen:

$= {}^8C_3 = \frac{8!}{3!\ 5!} = \frac{8 \times 7 \times 6 \times 5!}{5! \times 3 \times 2 \times 1}$

= 56 number of ways.

MISCELLANEOUS EXERCISE

Q-1: Find the number of words, having meaning or meaningless, each of 2 vowels and 4 consonants will be formed by the letters of the word DAUGHTER?

Solution:

In **DAUGHTER** word, we have **3 vowels** which is **A, U and E**, rest **5 letters** are **consonants** which are **D, G, H, T and R.**

Now,

The number of way to select **2 vowels** at a time from **3 vowels** will have **permutation** say, $^3C_2$ **= 3**

Also, the number of way to select **4 consonants** at a time from **5 consonants** will have **permutation** say, $^5C_4$= 5

Hence, the total number of combinations of **2 vowels** and **4 consonants** = 3 × 5 **= 15**

These **15 combinations** of **2 vowels** and **4 consonants** will be arranged in **6! number of ways.**

Therefore, the number of different words required = 15 × 5! = 1800

Q-2: Find the number of words, which can be either meaningful or meaningless, which will be formed by using all of the letters of the word EQUATION at a time such that both the vowel and the consonant will occur together.

Solution:

In the given word **EQUATION**, we have **5 vowels**, that are **E, U, A, I and O,** and there are **3 consonants**, that are **Q, T and N.**

As all the **vowels** and **consonants** will occur together, means both **(A, E, I, O, U) and (Q, T, N)** will be considered as a **single letter**.

Thus, the **permutation** of **vowels and consonants** taken all at a single time is counted which is: $^2P_2$ **= 2!**

Now, at each of the **permutation** for vowels, there is a **permutation of 5!** Taken

Now, at each of the **permutation** for vowels, there is a **permutation of 3!** taken all at a time and for consonants, the **permutation is 3!** taken all at a time.

Therefore, by the principle of multiplication, number of the words required = 2! × 5! × 3! = 1440

Q-3: There are 10 boys and 5 girls through which a committee with 7 members has to be formed. Now, find the number of ways in which this might be created when the committee formed consists of:

(i) Exactly 4 girls **(ii) at least 4 girls** **(iii) almost 4 girls?**

Solution:

(i) From 10 boys and 5 girls, a committee of 7 members is being formed.

As in this case, exactly **4 girls** is needed to be there in the committee, so each committee should contain (7 – 4) = **3 boys** in the committee.

Therefore, number of ways required in this case:

$= {}^5C_4 \times {}^{10}C_3$

$= \frac{5!}{4!\ 1!} \times \frac{10!}{3!\ 7!} = 5 \times \frac{10 \times 9 \times 8 \times 7!}{7! \times 3 \times 2 \times 1}$

= 600 number of ways

(ii) Here, in this case, at least **4 girls** can be there in each committee.

So, in such cases, a committee can consist of

(a) Either 3 boys or 4 girls **(b) either 2 boys or 5 girls.**

(a) 4 boys and 3 girls will be selected in $^{10}C_3 \times {}^5C_4$ number of ways.

(b) 2 boys and 5 girls will be selected in $^{10}C_2 \times {}^5C_5$ number of ways.

Hence, the number of ways required in this case:

$= {}^{10}C_3 \times {}^5C_4 + {}^{10}C_2 \times {}^5C_5$

$= \frac{10!}{3!\ 7!} \times \frac{5!}{4!\ 1!} + \frac{10!}{2!\ 8!} \times \frac{5!}{5!\ 0!}$

$= \left[\frac{10 \times 9 \times 8 \times 7!}{3 \times 2 \times 1 \times 7!} \times 5\right] + \left[\frac{10 \times 9 \times 8!}{2 \times 1 \times 8!} \times 1\right] = (10 \times 3 \times 4 \times 5) + (5 \times 9)$

= 645 number of ways.

(iii) Here, in this case, almost **4 girls** can be there in each of the committee.

So, in this case, a committee can consist of:

(a) 4 girls and 3 boys **(b) 3 girls and 4 boys**

(c) 2 girls and 5 boys **(d) 1 girl and 6 boys**

(e) No girl and 7 boys

(a) 4 girls and 3 boys will be selected in $^5C_4 \times {}^{10}C_3$ number of ways.

(b) 3 girls and 4 boys will be selected in $^5C_3 \times {}^{10}C_4$ number of ways.

(c) 2 girls and 5 boys will be selected in $^5C_2 \times {}^{10}C_5$ number of ways.

(d) 1 girl and 6 boys will be selected in $^5C_1 \times {}^{10}C_6$ number of ways.

(e) No girl and 7 boys will be selected in $^5C_0 \times {}^{10}C_7$ number of ways

Hence, the number of ways required in this case:

$= {}^5C_4 \times {}^{10}C_3 + {}^5C_3 \times {}^{10}C_4 + {}^5C_2 \times {}^{10}C_5 + {}^5C_1 \times {}^{10}C_6 + {}^5C_0 \times {}^{10}C_7$

$= \frac{5!}{4!\,1!} \times \frac{10!}{3!\,7!} + \frac{5!}{3!\,2!} \times \frac{10!}{4!\,6!} + \frac{5!}{2!\,3!} \times \frac{10!}{5!\,5!} + \frac{5!}{1!\,4!} \times \frac{10!}{6!\,4!} + \frac{5!}{0!\,5!} \times \frac{10!}{7!\,3!}$

= 300 + 2100 + 2520 + 1050 + 120

= 6090 number of ways.

Q-4: In a dictionary, the permutation of the letters of the word EXAMINATION is listed, then find the total number of words in the list of dictionary before the first letter of the word start with E.

Solution:

There are **11 letters** in the given word **EXAMINATION** where **A, I and N** appeared **twice** and all other letter appeared only for **once**.

Words which are listed before the word which is starting with **E** in a dictionary can be the word which starts with only **A**.

Hence, in order to get the numbers of **words starting from A**, means the **letter A** will be fixed at the extreme left position of the word, and there are **10** remaining letters taken at a time is rearranged.

In the given word which is **EXAMINATION**, there are **2 N's and 2 I's** in the remaining letters and other letters occurred only for once.

Thus, the number of words starting with the letter A:

$= \frac{10!}{2!\,2!}$ = 907200

Hence, the number of words required is 907200.

Q-5: Find the total number of 6- digit numbers formed by using the digits 0, 2, 3, 4, 5 and 6 which should be divisible by 10. Note that, the repetition of digit is not allowed.

Solution:

For a number to be **divisible by 10,** then its **unit place** must be **filled** with **0 only**.

Thus, **units** place is fixed which is to be filled only **by 0**.

Thus, **units** place is fixed which is to be filled only **by 0.**

Hence, there are multiple numbers of ways to fill up the other **5 vacant places** in succession with the other digits that is, **2, 3, 4, 5 and 6**.

Now,

Other **5 vacant places** will be filled in by **5! number of ways.**

Therefore, the number of 6-digit numbers required = 5! = 120

Q-6: It is known that the English alphabet has 21 consonants and 5 vowels. Find the number of words formed with 4 different consonants and 3 different vowels from the English alphabet.

Solution:

We need to select 4 different **consonants** from **21 consonants** and **3 different vowels** from **5 vowels** of the **English alphabet**.

So, the **permutation** of the selection of **vowels** is given by:

$^5C_3 = \frac{5!}{3!\ 2!} = 10$

Also, the permutation of the selection of consonants is given by:

$^{21}C_2 = \frac{21!}{4!\ 17!} = 3990$

Hence, the total number of the combinations of 4 different consonants and 3 different consonants = 10 × 3990 = 39900.

Q-7: There are 12 questions in the question paper of an examination which is mainly divided into two parts, say, part-i and part-ii, each having 4 and 8 questions, respectively. There is a condition given in the question paper that the student has to attempt at least 8 questions for sure, selection 3 from each section. Now, find the number of ways in which a student can select the questions in the question paper.

Solution:

From the data given in the question, know that the **examination question paper** is **divided** into **two parts**, namely; **part-I and part-II,** containing **4 and 8 questions each.**

A student need to attempt at least **8 questions**, selecting **3 questions** at least from **each sections**, which can be done in following ways:

(a) 4 questions from part I and 4 questions from part II

(b) 3 questions from part I and 5 questions from part II

The selection of **4 questions from part** I and **4 questions from part** II have permutation as: $^4C_4 \times {}^8C_4$ **number of ways.**

The selection of **3 questions** from **part I** and **5 questions** from **part II** have **permutation** as: $^4C_3 \times {}^8C_5$ **number of ways.**

Now,

The total number of ways in which selection of questions are required:

= $^4C_4 \times {^8C_4} + {^4C_3} \times {^8C_5}$

$= \frac{4!}{4!\ 0!} \times \frac{8!}{4!\ 4!} + \frac{4!}{3!\ 1!} \times \frac{8!}{5!\ 3!} = 1 + \frac{8 \times 7 \times 6 \times 5!}{5! \times 3 \times 2 \times 1}$

= 57 number of ways

Q-8: Find the number of ways of 5- card combinations from a deck of 52 cards. Note that, among those 5 cards, there must be exactly one king in the combination.

Solution:

We need to make **5 – card** combinations from a deck of **52 cards**, in such a way that there must be a **king** in the combination.

We know that, there are **4 kings** in a deck of **52 cards**.

Then,

The **permutation** of the selection of a **king** from the **deck** is: $^4C_1$ **number of ways.**

Now after selection of a **king** for the **combination** of **5 cards** then, there are **4 cards** left to be selected.

Thus, **4 cards** can be selected from the deck with **48 cards** left in $^{48}C_4$ **number of ways.**

Therefore, the number of 5- card combination required is:

$= {^4C_1} \times {^{48}C_4} = \frac{4!}{1!\ 3!} \times \frac{48!}{4!\ 44!} = \frac{48 \times 47 \times 46 \times 45 \times 44!}{3 \times 2 \times 1 \times 44!}$

= 7,78,320 number of combinations.

Q-9: Consider a situation in which 7 men and 6 women needs to be seated in a row in such a way that the women will occupy the even places.

Solution:

It is required to arrange the seating of **7 men** and **6 women** in a row in such a way that the women will occupy the **even places**.

7 men can be seated in **7! Ways**. For every arrangement, **6 women** will be seated only at the **blank (_) places** so that the **women** can occupy the **even places.**

A _ A _ A _ A _ A _ A_ A

Thus, the **women** will be **seated** in **6! number** of **ways**.

Therefore, Number of arrangements possible as per the given condition = 6! × 7! = 3628800 number of arrangements.

Q-10: There are 20 students in a class from which 8 students will be chosen for an exclusion party. 3 students from the class decided that either they will join combinely or none of them will go for the exclusion party. Find the number of ways by which the exclusion party will be chosen.

Solution:

8 students will be selected for the **exclusion party** from a class of **20 students**.

Also, it is given in the question that, **3 students** from that class decided that they will **either** go **together or none of them will go for the exclusion party**. Thus, there are mainly 2 cases:

(a) All 3 of them are going for the exclusion party.

Thus, the remaining **5 students** will be selected from the remaining **17 students** in the class in $^{17}C_5$ **number of ways.**

(b) None of them are going for the exclusion party.

Thus, 8 **students** will be selected from the class of **20 students** in $^{20}C_8$ **number of ways**.

Therefore, the **number of ways** required for **choosing students** for the **exclusion party** is:

$= {}^{17}C_5 + {}^{20}C_8 = \frac{17!}{5!\ 12!} \times \frac{20!}{8!\ 12!}$

$= \frac{17 \times 16 \times 15 \times 14 \times 13 \times 12!}{5 \times 4 \times 3 \times 2 \times 1 \times 12!} + \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12!}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 12!}$ **= 6188 + 125970**

= 132158 number of ways to choose the students for the exclusion party.

Q-11: Arrange ASSASSINATION in such a way that all the S's are filled together. Find the number ways for the arrangement of S's together.

Solution:

ASSASSINATION is the word given in the question where **A** appeared for **3 times**, **S** appeared for **4 times**, **I** appeared for **2 times**, **N** appeared for **2 times** and, **T and O** appeared just for **once**.

We need to arrange all the words in such a way that the all **S**'s are **placed** together.

Now,

SSSS is considered as a **single object**.

Since, including the single object we have remaining **9 letters**, so we have the count of **10 letters** more.

Thus, the **10 letters** where they have **3 A's, 2 I's and 2 N's** will be arranged in $\frac{10!}{3!\ 2!\ 2!}$ **number of ways.**

Therefore, the total number of ways required to arrange the letters of

Therefore, the total number of ways required to arrange the letters of ASSASSINATION word = $\frac{10!}{3!\,2!\,2!}$ = 151200