NCERT SOLUTIONS
CLASS-XI PHYSICS
CHAPTER-14
OSCILLATIONS AND RESONANCE

Q1. Which of the following is periodic motions:
(a) A swimmer making to and fro laps in a river.
(b) A star revolving around a black hole.
(c) A bullet fired from a gun.
(d) A freely suspended bar magnet is moved away from its N –S direction and released.

ANSWER:

(a) A swimmer's motion maybe to and fro but as it lacks a definite period, it is not periodic.
(b) A star revolving a black hole has a periodic motion as it always returns back to the same position after a definite amount of time.
(c) A bullet fired from a gun isn't periodic as it does not return back to the gun.
(d) When a freely suspended magnet is moved out from its N-S direction its motion will be periodic, because it oscillates about its mean position within definite intervals of time

Q2. Identify the ones with simple harmonic motion and the ones with periodic motion but not simple harmonic:
(a) rotation of Pluto about its axis.
(b) oscillating mercury column in a u-tube.
(c) a ball bearing being released from slightly above the bowl's lowermost point.
(d) a polyatomic molecule's general vibration about its equilibrium position.

ANSWER:

(a) It is a periodic motion but not simple harmonic motion because there is no to and fro motion about a fixed point.
(b) It is simple harmonic motion.
(c) It is simple harmonic motion.
(d) A polyatomic molecule's general vibration is the superposition of individual simple harmonic motions of a number of different molecules. Hence, it is not simple harmonic, but periodic

Q3. Among the four x-t plots for linear motion of a particle, identify the graphs that represent a periodic motion. Also, find the find the period of motion if it is periodic.

ANSWER:

(a) It is non-periodic. As motion is not repeated.

(b) As the motion is repeated every 2 seconds, it is a periodic motion. Period = 2s.

(c) Non-periodic as motion is repeated in one position only.

(d) As the motion is repeated every 2 seconds, it is a periodic motion. Period = 2s.

Q4. Among the given functions of time which are the ones representing:
(i) simple harmonic, (ii) periodic but not simple harmonic motion, and (iii) non-periodic motion?
Find the period for each case of periodic motion (ω is a positive constant):
a) sin ωt – cos ωt
b) $\sin^3 \omega t$
c) 3 cos (π/4 – 2ωt)
d) cos ωt + cos 3ωt + cos 5ωt
e) $\exp(-\omega^2 t^2)$
f) $1 + \omega t + \omega^2 t^2$

ANSWER:

(a) sin ωt – cos ωt
$= \sqrt{2}[\frac{1}{\sqrt{2}} sin\omega t - \frac{1}{\sqrt{2}} cos\omega t]$

$= \sqrt{2}[cos\frac{\pi}{4} \times sin\omega t - sin\frac{\pi}{4} \times cos\omega t]$
$= \sqrt{2} sin[\omega t - \frac{\pi}{4}]$

As this can be represented as asin (ωt + Φ) it represents SIMPLE HARMONIC MOTION
Its period is : $\frac{2\pi}{\omega}$

(b) $\sin^3 \omega t$ = ¼ [3sin ωt –sin 3ωt]
Even though the two sinωt represent simple harmonic motions respectively, but they are periodic because superposition of two SIMPLE HARMONIC MOTION is not simple harmonic.

(c) 3 cos (π/4 – 2ωt) = 3 cos (2ωt – π/4)
As it can be written as : a sin (ωt + Φ) , it represents SIMPLE HARMONIC MOTION
Its period is : π/ω

(d) In cos ωt + cos 3ωt + cos 5ωt, each cosine function represents SIMPLE HARMONIC MOTION, but the super position of SIMPLE HARMONIC MOTION gives periodic.

(e) As it is an exponential function, it is non periodic as it does not repeat itself.

(f) $1 + \omega t + \omega^2 t^2$ is non periodic.

Q5. A body having linear simple harmonic motion between two points, X and Y, 20 cm apart. If direction from X to Y is the positive direction, then provide signs of acceleration, force, and velocity on the body when it is
(i) at X,
(ii) at Y,
(iii) at the mid-point of XY moving towards X,
(iv) at 2 cm away from Y moving towards X,
(v) at 3 cm away from X moving towards Y,
(vi) at 4 cm away from Y moving towards X.

ANSWER:

(i) At X the body with Simple harmonic motion is momentarily at rest, thus its velocity is zero. Force and accelerations are positive as it is directed towards Y from X.
(ii) At Y velocity is zero, force and accelerations are negative as they are directed towards X from Y.
(iii) At the midpoint of XY moving towards X, the body has positive velocity but negative acceleration and force.
(iv) When the body is 2 cm away from Y moving towards X it has positive velocity but negative acceleration and force.
(v) When the body is 3 cm away from X moving towards Y, it has positive force, acceleration, and velocity.
(vi) When the body is 4 cm away from Y moving towards X it has positive velocity but negative acceleration and force.

Q6. Which among the following relationships between displacement, 's' and acceleration 'a' of a body represents simple harmonic motion?
a) a = 0.5s
b) a = –10s
c) a = –10s +5
d) a = 300s^3

ANSWER: For

For the simple harmonic motion to be present the requisite relation between acceleration and displacement is:
a = –k s. Which is being satisfied by the relation in option (b).

Q7. The motion of a body in simple harmonic motion is given by the displacement function,
x (t) = A cos (ωt + φ).
Given that at t = 0, the initial velocity of the body is ω cm/s and its initial position is 1 cm, calculate its initial phase angle and amplitude?
If in place of the cosine function, a sine function is used to represent the simple harmonic motion: x = B sin (ωt + α), calculate the body's amplitude and initial phase considering the initial conditions given above. [Angular frequency of the particle is π/ s]

ANSWER:

Given,
Initially, at t = 0:
Displacement, x = 1 cm
Initial velocity, v = ω cm/sec.
Angular frequency, $\omega = \pi$ rad/s

It is given that:
x(t) = A cos(ωt + Φ) . . . . (i)

1 = A cos(ω x 0 + Φ) = Acos Φ
A cos Φ = 1 . . . . (ii)

Velocity, v = dx / dt
differentiating equation (i) w.r.t 't'
v = – Aωsin (ωt + Φ)

Now at t = 0; v = ω and
=> ω = – Aωsin (ωt + Φ)

$1 = -A \sin(\omega \times 0 + \Phi) = -A\sin(\Phi)$
$A\sin(\Phi) = -1$ . . . . (iii)

Adding and squaring equations (ii) and (iii), we get:
$A^2(\sin^2 \Phi + \cos^2 \Phi) = 1 + 1$
thus, $A = \sqrt{2}$

Dividing equation (iii) by (ii), we get :
$\tan \Phi = -1$
Thus, $\Phi = 3\pi/4$, $7\pi/4$

Now if SIMPLE HARMONIC MOTION is given as :
$x = B \sin(\omega t + \alpha)$

Putting the given values in the equation , we get :
$1 = B \sin(\omega \times 0 + \alpha)$
$B\sin \alpha = 1$ . . . . (iv)

Also, velocity (v) = $\omega B\cos(\omega t + \alpha)$
Substituting the values we get :
$\pi = \pi B\sin \alpha$
$B\sin \alpha = 1$ . . . . (v)

Adding and squaring equations (iv) and (v), we get:
$B^2[\sin^2 \alpha + \cos^2 \alpha] = 2$
Therefore, $B = \sqrt{2}$

Dividing equation (iv) by equation (v), we get :

$B\sin \alpha / B\cos \alpha = 1$
$\tan\alpha = 1 = \tan(\pi/4)$
Therefore, $\alpha = \pi/4$, $5\pi/4$, ...

Q8. A spring balance has a scale with the range of 0 to 100 kg. An object suspended from this balance, when displaced and released, starts oscillating with a period of 0.6 s. Find the weight of this object?
[Length of the scale is 40 cm].

ANSWER:

Given,
Maximum mass that the scale can read, M = 100 kg.
Maximum displacement of the spring = Length of the scale, l = 40 cm = 0.4 m
Time period, T = 0.6 s

We know,
Maximum force exerted on the spring, F = Mg
Where, g = 9.8 m/s^2
=> F = 100 × 9.8 = 980 N

Thus, Spring constant, k = F / l
k = 980/0.4 = 2450 N/m

Now,
Let the object have a mass m.
We know,
Time period, $t = 2\pi\sqrt{\frac{m}{k}}$
$m = \left(\frac{t}{2\pi}\right)^2 \times k$
$m = \left(\frac{0.6}{2\times 3.14}\right)^2 \times 2450$
Therefore, $m = 22.36kg$

Thus, the weight of the object is 22.36 x 9.8 = 219.167 N

Q9. A spring with a spring constant of 1200 N / m is placed on a horizontal plane as depicted in the figure below. A 6 kg mass is then hooked to the free end of the spring. The mass is then pulled rightwards for 4 cm and released. Calculate (i) Oscillation frequency, (ii) Maximum acceleration of the mass, and (iii) Maximum speed of the mass.

Sol:

Given,
Spring constant, k = 1200 N/m
Mass, m = 6 kg
Displacement, A = 4.0 cm = 0.04 cm

(i) Oscillation frequency v = 1/T
Where, T = time period.
$v = \frac{1}{2\times\pi}\sqrt{\frac{k}{m}}$
Therefore, $v = \frac{1}{2\times 3.14}\sqrt{\frac{1200}{6}}$
= 2.25 per second

(ii) Maximum acceleration (a) = ω^2 A
Where,
ω = Angular frequency = $\sqrt{\frac{k}{m}}$
A = maximum displacement

Therefore, a = A(k/m)
a = 0.04 x (1200/6) = 8 m s^2

(iii) Maximum velocity, $V_{MAX} = \omega$ A
= 0.04 X $\sqrt{\frac{1200}{6}}$
Therefore, V_{MAX} = 0.56 m / s

Q10. In the above question let us consider the position of mass when the spring is relaxed as x = 0, and the left to right direction as the positive direction of the x-axis.
Provide x as a function of time t for the oscillating mass, if at the moment we start the stopwatch (t = 0), the mass is:
(i) at the mean position,
(ii) at the maximum stretched position, and
(iii) at the maximum compressed position.

Sol:

Given,
Spring constant, k = 1200 N/m
Mass, m = 6 kg
Displacement, A = 4.0 cm = 0.04 cm
ω = 14.14 s^{-1}
(i)Since time is measured from mean position,
x = A sin ω t
x = 4 sin 14.14t

(ii) At the maximum stretched position, the mass has an initial phase of π/2 rad.
Then, x = A sin(ωt + π/2) = A cos ωt
= 4 cos 14.14t

(iii) At the maximum compressed position, the mass is at its leftmost position with an initial phase of 3π/2 rad. Then, x = A sin(ωt + 3π/2) = -4 cos14.14 t

Q11.Following diagrams represent two circular motions. The revolution period, the radius of the circle, the sense of revolution (i.e. clockwise or anticlockwise) and the initial position, are indicated on each diagram.
Find the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving body B, in each situation.

Sol:

(1) For time period, T = 4 s
Amplitude, A = 3 cm
At time, t = 0, the radius vector OB makes an angle π/2 with the positive x-axis, i.e.,
Phase angel Φ = + π/2

Therfore, the equation of simple harmonic motion for the x-projection of OB, at time t is:
x = A cos [2πt/T + Φ]
= 3 cos [2πt/4 + π/2] = -3sin (πt/2)
= -3sin (πt/2) cm

(2) Time period, T = 8 s
Amplitude, A = 2 m
At time t = 0, OB makes an angle π with the x-axis, in the anticlockwise direction.
Thus, phase angle, Φ = + π
Therefore, the equation of simple harmonic motion for the x-projection of OB, at time t is:
x = A cos [2πt/T + Φ]
= 2 cos [2πt/8 + π]
= -2 cos (πt/4)

Q12. Draw the corresponding reference circle for each of the following simple harmonic motions. Mention the initial (t = 0) position of the body, the angular speed of the rotating body and the radius of the circle. For simplification, consider the sense of rotation to be anticlockwise in each case: [x is in cm and t is in s].
(i) x = –2 sin (3t + π/3)
(ii) x = cos (π/6 – t)
(iii) x = 3 sin (2πt + π/4)
(iv) x = 2 cos πt

Sol:

(a) x = –2 sin (3t + π/3) = 2cos(3t + π/3 + π/2)
= 2 cos (3t + 5π/6)

On comparing this equation with the standard equation for Simple harmonic motion:
x = A cos [2πt/T + Φ]

We get,
Amplitude = 2 cm
Phase angle = 5π/6 = 150^0
Angular velocity= ω = 2πt/T = 3 rad/sec
Thus the corresponding reference circle for motion of this body is as :
(ii) x = cos (π/6 – t)
= cos (t – π/6)
On comparing this equation with the standard equation for Simple harmonic motion ;
x = A cos [2πt/T + Φ]

We get,
Amplitude = 1 cm
Phase angle = -π/6 = -30^0
Angular velocity= ω = 2πt/T = 1 rad/sec

Thus the corresponding reference circle for motion of this body is as:

(iii) x = 3 sin (2πt + π/4)
= – 3 cos [(2πt + π/4) + π/2] = -3cos(2πt + 3π/4)
On comparing this equation with the standard equation for Simple harmonic motion ;
x = A cos [2πt/T + Φ]

We get;
Amplitude = 3 cm
Phase angle = 3π/4 = 135^0
Angular velocity= ω = 2πt/T = 2 rad/sec

Thus the corresponding reference circle for motion of this body is as:

(iv) x = 2 cos πt
On comparing this equation with the standard equation for Simple harmonic motion:
x = A cos [2πt/T + Φ]

We get,
Amplitude = 2 cm
Phase angle = 0^0
Angular velocity= ω = π rad/sec
Thus the corresponding reference circle for motion of this body is as:

Q13.Figure below (a) depicts a spring of force constant k attached at one end and a block of mass, 'm' clamped to its free end. A force F is applied on the free end of the spring to stretch it. Figure (b) depicts the same spring with both of its ends clamped to blocks of mass m. Both of the ends of the spring in figure (b) is stretched by the same force F.
Find (i) the spring's maximum extension in both the cases.
(ii) If the block in figure (a) and the two blocks in figure (b) are released, what would the oscillation period be in the two cases?

Sol:
For the one block system:
When force F, is applied to the free end of the spring, there is an extension.

(i)For the maximum extension, we know: F = kl
Where,
k is the spring constant.
Thus, the maximum extension produced in the spring ,l = F/k

For the two block system:
The displacement (x) produced in this case is:
x = l/2
Net force, F = 2 k x [l =2x]
=> F = 2k (l/2)

Therefore, l = F / k

(ii) For one block system :

Force on the block of mass m , F = ma = $\frac{md^2x}{dt^2}$

Where, x is the displacement of the block in time t

Therefore, $\frac{md^2x}{dt^2}$ = -kx

The negative sign is present as the direction of the elastic force is opposite to the direction of the displacement.

$\frac{d^2x}{dt^2}$ = – x (k/m) = – ω^2 x

Where, ω^2 =k/m

ω is the angular frequency of oscillation.

Therefore, time period of oscillation, T = 2π/ω

= 2π(m/k)$^{1/2}$

For two block system :

F = ma = $\frac{md^2x}{dt^2}$

$\frac{md^2x}{dt^2}$ = -2kx

The negative sign is present as the direction of the elastic force is opposite to the direction of the displacement.

$\frac{d^2x}{dt^2}$ = – 2x (k/m) = – ω^2 x

Where, ω^2 =2k/m

Where, ω is the angular frequency of oscillation.

Therefore, time period of oscillation, T = 2π/ω

= 2π (m/2k)$^{1/2}$

Q14. The piston in the cylinder head of a V8 engine has a stroke (twice the amplitude) of 2.0 m. Given that the piston moves with simple harmonic motion at an angular frequency of 400 rad/min, find its maximum speed?

Sol:

Given,

The angular frequency of the piston, ω = 400 rad/ min.

Stroke = 2.0 m

Amplitude, A = 2/2 = 1 m

Thus, maximum speed V_{MAX} =A ω = 400 m/ min

Q15. On the surface of the moon, the value acceleration due to gravity is 1.7 ms^{-2}. Calculate the time period of a simple pendulum on the lunar surface if its time period on earth is 7 s?

Sol:

Given,

Acceleration due to gravity on the surface of moon, g' = 1.7 m s^{-2}

Acceleration due to gravity on the surface of earth, g = 9.8 m s^{-2}

Time period of a simple pendulum on earth, T = 7s

We know,

$T = 2\pi\sqrt{\frac{l}{g}}$

Where, l is the length of the pendulum

$l = \frac{T^2}{(2\pi)^2} \times 9.8$

= 12.17m

On the moon's surface, time period T' =$2\pi\sqrt{\frac{l}{g'}}$

Therefore, $T' = 2\pi\sqrt{\frac{12.17}{1.7'}}$ = 16.82 s

Q16. (i) The time period of a body having simple harmonic motion depends on the mass m of the body and the force constant k:

T =$2\pi\sqrt{\frac{m}{k}}$

A simple pendulum exhibits simple harmonic motion. Then why does the time period of a pendulum not depend upon its mass?

(ii) For small angle oscillations, a simple pendulum exhibits simple harmonic motion (more or less). For larger angles of oscillation, detailed analysis show that T is greater than $2\pi\sqrt{\frac{l}{g}}$. Explain.

(iii) A boy with a wristwatch on his hand jumps from a helicopter. Will the wrist watch give the correct time during free fall?
(iv) Find the frequency of oscillation of a simple pendulum that is free falling from a tall bridge.

Sol:

(i) The time period of a simple pendulum, T $=2\pi\sqrt{\frac{m}{k}}$

For a simple pendulum, k is expressed in terms of mass m, as:
k ∝ m
m/k = constant
Thus, the time period T, of a simple pendulum is independent of its mass.

(ii) In the case of a simple pendulum, the restoring force acting on the bob of the pendulum is:
F = –mg sinθ
Where,
F = Restoring force
m = Mass of the bob
g = Acceleration due to gravity
θ = Angle of displacement
For small θ, sin θ ~ θ
For large θ, sin θ is greater than θ. This decreases the effective value of g.
Thus, the time period increases as:
T $=2\pi\sqrt{\frac{l}{g'}}$

(iii) As the working of a wrist watch does not depend upon the acceleration due to gravity, the time shown by it will be correct during free fall.
(iv) As acceleration due to gravity is zero during free fall, the frequency of oscillation will also be zero.

Q17. A simple pendulum with a bob of mass M and a pendulum of length l is suspended in a car. The car then moves on a circular track of radius R at a constant velocity v. If the pendulum makes small oscillations in a radial direction from its equilibrium position, find its time period.

Sol:

The bob of the simple pendulum will experience centripetal acceleration because by the circular motion of the car and the acceleration due to gravity.
Acceleration due to gravity = g
Centripetal acceleration = v^2 / R
Where,
v is the uniform speed of the car
R is the radius of the track
Effective acceleration (a_{aeff}) is given as :
a_{aeff} = $\sqrt{g^2+(\frac{v^2}{R})^2}$

Time period, T = $2\pi\sqrt{\frac{l}{a_{aeff}}}$

Where, l is the length of the pendulum
Therefore, Time period T = $2\pi\sqrt{\frac{l}{\sqrt{g^2+\frac{v^4}{R^2}}}}$

Q18. A cylindrical cork of base area A, height h and of density ρ floats in a liquid of density ρ_1. The cork is pushed into the liquid slightly and then released. Prove that the cork oscillates up and down with simple harmonic motion at a period of T = $2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$

Sol:

Given,
Base area of the cork = A
Height of the cork = h
Density of the liquid = ρ_1
Density of the cork = ρ

In equilibrium:
The weight of the cork = Weight of the liquid displaced by the floating cork.

Let the cork be dipped slightly by x.
As a result, some excess water of a certain volume is displaced. Thus, an extra up-thrust provides the restoring force to the cork.
Up-thrust = Restoring force,
F = Weight of the extra water displaced
F = – (Volume × Density × g)
Volume = Area × Distance through which the cork is depressed
Volume = Ax

$F = -A \times \rho_1 g$ . . . . (i)

According to the force law:
F = kx
k = F/x
Where, k is a constant.
$k = F/x = -A \times \rho_1 g$ . . . . (ii)

The time period of oscillations of the cork :
$T = 2\pi\sqrt{\frac{m}{k}}$ . . . . (iii)
Where,
m = mass of the cork
= Volume x density
= Base area of the cork × Height of the cork × Density of the cork
= Ahρ

Therefore, the expression for the time period becomes :
$T = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$ . . . . . [Proved]

Q19. In a U-tube holding liquid mercury one end is open to the atmosphere and the other end is connected to a suction pump. If some pressure difference is maintained between the two columns. Prove that when the pump is removed, the mercury column in the U-tube exhibits simple harmonic motion.

Sol:

Area of cross-section of the U-tube = A
Density of the mercury column = ρ
Acceleration due to gravity = g
Restoring force, F = Weight of the column of mercury of a certain height
F = –(Volume × Density × g)
F = –(A × 2h × ρ ×g)
= –2Aρgh
= –k × Displacement in one arm (h)
Where,
2h is the height of the mercury column in the two arms
k is a constant,
given by, k = -F/h = 2Aρg

Time period,
$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2A\rho g}}$

Where,
m, is the mass of the mercury column
Let, l be the length of the total mercury in the U-tube.
Mass of mercury, m = Volume of mercury × Density of mercury = Alρ

Therefore, $T = 2\pi\sqrt{\frac{Al\rho}{2A\rho g}}$

i.e. $T = 2\pi\sqrt{\frac{l}{2g}}$

Thus, the mercury column exhibits SIMPLE HARMONIC MOTION with a time period of $2\pi\sqrt{\frac{l}{2g}}$