NCERT SOLUTIONS
CLASS-XI PHYSICS
CHAPTER-8
GRAVITATION

Q.1: (a). By putting an electric charge inside a hollow conductor you can shield it from the influences of electrical forces. Can a body be shielded from the gravitational influences of nearby matter in a similar fashion?
(b). An astronaut in a shuttle orbiting Mars cannot feel gravity. If he was traveling in a very large space ship would he have detected gravity?

(c). The Sun's gravitational force on Earth is greater than the moon's. However, the Moon affects the tidal waves much more than the Sun. Why?

Sol: (a). No, as of now, no method has been devised to shield a body from gravity because gravity is independent of medium and it is the virtue of each and every matter. So the shield would exert the gravitational forces

(b). Yes, if the spaceship is large enough then the astronaut will definitely detect the Mars gravity.

(c). Gravitational force is inversely proportional to the square of the distance whereas, **Tidal effects** are **inversely proportional** to the **cube** of the **distance**. So as the distance between the earth and moon is smaller than the distance between earth and sun, the moon will have a greater influence on the earth's **tidal waves**

Q2. Pick the right choice:
(a). Acceleration due to gravity does not depend upon the mass of the [body/earth]

(b). The formula, mg ($r_2 - r_1$) is [more/less] accurate than the formula $GMm\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$ for the difference of potential energy between two points at a distance of r_1 and r_2 from the earth's center.
(c) Acceleration due to gravity [decreases/increases] with increasing depth. (Consider earth has a uniform density).

(d) Acceleration due to gravity [decreases/increases] with decreasing altitude
Sol:

(a). body.

(b). more.

(c). decreases.

(d). increases.

Q.3: Imagine a planet that revolves around the sun at a double speed of the earth. Find the size of its orbit as compared to the orbit of the earth.

Sol:

Time taken by the earth for one complete **revolution**, T_E = **1 Year**

Radius of Earth's orbit, R_E = **1 AU**

Thus, the time taken by the planet to complete one complete revolution:

$T_P = \frac{1}{2}T_E = \frac{1}{2}$ year

Let, the orbital radius of this planet = R_P

Now, according to the **Kepler's third law of planetary motion**:

$$\left(\frac{R_P}{R_E}\right)^3 = \left(\frac{T_P}{T_E}\right)^2$$

$$\frac{R_P}{R_E} = \left(\frac{T_P}{T_E}\right)^{\frac{2}{3}}$$

$$\frac{R_P}{R_E} = \left(\frac{T_P}{T_E}\right)^{\frac{2}{3}} = \left(\frac{\frac{1}{2}T_E}{T_E}\right)^{\frac{2}{3}} = \left(\frac{1}{2}\right)^{\frac{2}{3}} = 0.63$$

Therefore, radius of orbit of this planet is 0.63 times smaller than the radius of orbit of the Earth.

Q.4: One of Jupiter's satellites, Io has an orbital radius of 4.22 x 10^8 m and its revolution time around Jupiter is 1.769 days. Prove that the mass of the Sun is a thousand times that of Jupiter.

Sol:

Given,

Orbital period of Io, T_{I0} = 1.769 days **= 1.769 × 24 × 60 × 60 s**

Orbital radius of Io, R_{I0} = 4.22 × 10^8 m

We know mass of Jupiter:

$M_J = 4\pi^2 R_{I0}^3 / G T_{I0}^2$ . . . . (1)

Where;

M_J = Mass of Jupiter

G = Universal gravitational constant

Also,

Orbital period of the earth,

T_E = 365.25 days = 365.25 × 24 × 60 × 60 s

Orbital radius of the Earth, R_E = 1 AU = 1.496 × 10^{11} m

We know that the mass of sun is:

$M_S = \frac{4\pi^2 R_E^3}{G T_E^2}$ . . . . (2)

Therefore, $\frac{M_S}{M_J} = \frac{4\pi^2 R_E^3}{G T_E^2} \times \frac{G T_{I0}^2}{4\pi^2 R_{I0}^3} = \frac{R_E^3}{T_E^2} \times \frac{T_{I0}^2}{R_{I0}^3}$

Now, on substituting the values, we will get:

$= \left[\frac{1.769\times24\times60\times60}{365.25\times24\times60\times60}\right]^2 \times \left[\frac{1.496\times10^{11}}{4.22\times10^8}\right]^3 = 1045.04$

Therefore, $\frac{M_S}{M_J} \sim 1000$

$M_S \sim 1000 \times M_J$

[Which proves that, the Sun's mass is 1000 times that of Jupiter's)

Q.5: Consider that the Sombrero galaxy has 3 x 10^{11} stars, each of one solar mass. If the diameter of the galaxy is 10^5 light years, calculate the time of a star, at a distance of 5000 light years from the galactic center, takes to complete one revolution.

Sol:

Mass of Sombrero Galaxy, M **= 3 × 10^{11} solar mass**
Solar mass = Mass of Sun = **2.0 × 10^{36} kg**
Mass of the galaxy, **M =** 3 × 10^{11} × 2 × 10^{36} **= 6 × 10^{41} kg**
Diameter of Sombrero Galaxy, **d = 5 x 10^4 ly**
Radius of Sombrero Galaxy, **r = 2.5 × 10^4 ly**

We know that:

1 light year = 9.46 × 10^{15} m
Therefore, r = 2.5 × 10^4 × 9.46 × 10^{15} **= 2.365 x 10^{20} m**

As this star revolves around the massive black hole in center of the Sombrero galaxy, its time period can be found with the relation:

$T = \left[\frac{4\pi^2 r^3}{GM}\right]^{\frac{1}{2}}$

$= \left[\frac{4\times3.14^2\times(2.365\times10^{20})^3}{(6.67\times10^{-11})\times(6\times10^{41})}\right]^{\frac{1}{2}}$ **= 4.246 x 10^{15} s**

Now we know, 1 year = 365 × 24 × 60 × 60 s

$1s = \frac{1}{365\times24\times60\times60}$ years

Therefore, 4.246 × 10^{15}s = $\frac{4.246\times10^{15}}{365\times24\times60\times60}$ = 1.34 × 10^8 years.

Q.6: Pick the right alternative:

(a). The energy spent on sending a rocket vertically upwards out of the earth's gravitational influence is **more / less** than the energy spent on sending an orbiting satellite to the same height out of the earth's influence.

(b). If potential energy is zero at infinity, the total energy of an orbiting comet is negative of its **potential / kinetic** energy.

Sol:

(a). more

(b). kinetic.

Q7. Choose the factors upon, which the escape velocity of an object from earth depends upon:

(a) body's mass.

(b) location of projection.

(c) direction of projection.

Sol: (b)

Explanation:

Escape velocity is independent of the direction of projection and the mass of the body. It depends upon the gravitational potential at the place from where the body is projected. **Gravitational potential depends slightly on the altitude and the latitude** of the place, thus **escape velocity depends slightly upon the location from where it is projected.**

Q.8: An asteroid orbits a star in an elliptical orbit. Does the asteroid have a constant (a) angular speed, (b) kinetic energy, (c) total energy (d), linear speed, (e) potential energy and (f) angular momentum throughout its revolution?

Sol:

An asteroid orbiting a star will have constant angular momentum and the constant value of total energy throughout its orbit.

Q9. Which of the following problems is likely to affect an astronaut in space? (a) Swollen feet, (b) bone loss, (c) orientational problem, (d) headache.

Sol:

(a). In zero gravity the blood flow to the feet isn't increased so the astronaut does not get **swollen feet.**

(b). Due to zero gravity, the weight the bones have to bear is greatly reduced, this causes **bone loss** in astronauts spending greater amounts of time in space.

(c). Space has different orientations, so **orientational problems** can affect an astronaut.

(d). Due to increased blood supply to their faces, astronauts can be affected by **headaches.**

Q.10: Pick up the right answers from the given ones:

The direction of gravitational intensity inside a hemispherical sphere of a uniform mass is indicated by the arrow:

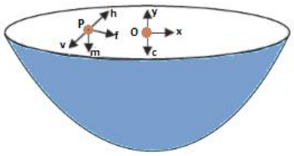
(i) c

(ii) x

(ii) x

(iii) z

(iv) o



Sol: (i) c

Reason:

Inside a hollow sphere, gravitational forces on any particle at any point is symmetrically placed. However, in this case, the upper half of the sphere is removed. Since gravitational intensity is gravitational force per unit mass it will act in direction point **downwards along 'c'.**

Q.11: For the above problem the gravitational intensity at P will be directed towards:

(i) h

(ii) f

(iii) m

(iv) v

Sol: (iii) m

Reason: Making use of the logic/explanation from the above answer we can conclude that the **gravitational intensity at P is directed downwards along m.**

Q12. A rocket is launched off from the earth's surface for a massive comet nearing the earth. At what distance from the earth's center will the shuttle of the rocket be free from the earth's gravitational force. Mass of the comet = 7.4 x 10^{22}, mass of Earth = 6 x 10^{24} and distance between earth and the comet= 3.84 x 10^{10} m.

Sol:

Given:

Mass of the comet, M_C = 7.4 × 10^{24} kg

Mass of the Earth, M_E = 6 × 10^{24} kg
Orbital radius, r = 3.84 × 10^{10} m
Mass of the shuttle = m kg

Let **'x'** be the **distance** from the center of the **Earth** where the **gravitational force** acting on the Shuttle **'S'** becomes **zero**.

According to **Newton's law of gravitation**, we have:

$$\Rightarrow \frac{G\ m\ M_A}{(r-x)^2} = \frac{G\ m\ M_E}{x^2}$$

$$\Rightarrow \frac{M_A}{M_E} = \left(\frac{r-x}{x}\right)^2$$

$$\Rightarrow \frac{r}{x} - 1 = \left(\frac{7.4\times10^{24}}{6\times10^{24}}\right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{r}{x} - 1 = 1.1106$$

$$\Rightarrow x = \frac{r}{2.1106} = \frac{3.84\times10^{10}}{2.1106} = 1.819 \times 10^{10}\ m$$

Q.13: If the earth's orbit around the sun is 1.5x 10^8 km long, estimate the mass of the Sun.

Sol:

Given:

Earth's orbit, r = 1.5 × 10^{11} m

Time taken by the Earth for one complete revolution,

T = 1 year = **365.25 days**

i.e. **T = (365.25 × 24 × 60 × 60) seconds**

Since, Universal gravitational constant, G = 6.67 × 10^{-11} Nm^2 kg^{-2}

Therefore, **mass of the Sun, M =** $\frac{4\,\pi^2 r^3}{G\,T^2}$

$$\Rightarrow M = \frac{4\times(3.14)^2\times(1.5\times10^{11})^3}{(6.67\times10^{-11})\times(365.25\times24\times60\times60)^2}$$

$$\Rightarrow M = \frac{4\times(3.14)^2\times(1.5\times10^{11})^3}{(6.67\times10^{-11})\times(365.25\times24\times60\times60)^2}$$

$$\Rightarrow \quad M = \frac{1.331\times10^{35}}{66.425\times10^{3}} = 2.004 \times 10^{30}\ kg$$

Therefore, the estimated mass of the Sun is 2.004 × 10^{30} Kg

Q14. Uranus is 84 times the earth year. If the Earth is 15 x 10^7 km away from the Sun, what is the distance between the Sun and Uranus?

Sol:

Given:

Distance between Earth and the Sun, **r_e =** 15 × 10^7 km **= 1.5 × 10^{11} m**

Time period of the Earth **= T_e**

Time period of Uranus, **T_u = 84 T_e**

Let, the **distance between** the **Sun** and the **Uranus** be **r_u**

Now, according to the **Kepler's third law of planetary motion**:

T = $\left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}}$

For **Uranus** and **Sun**, we can write:

$$\Rightarrow \frac{r_u^3}{r_e^3} = \frac{T_u^2}{T_e^2}$$

$$\Rightarrow r_u = r_e\left[\frac{T_u}{T_e}\right]^{\frac{2}{3}}$$

$$\Rightarrow r_u = 1.5 \times 10^{11}\left[\frac{84\ T_e}{T_e}\right]^{\frac{2}{3}} = \mathbf{1.5 \times 10^{11} \times (84)^{\frac{2}{3}}}$$

$$\Rightarrow r_u = 28.77 \times 10^{12}\text{m}$$

Therefore, Uranus is 28.77 × 10^{12} m away from the Sun.

Q.15: A man weighing 60 N on the earth's surface is taken to the height that is equal to half of the earth's radius. What is the amount of gravitational force which the earth will exert on this man now?

Sol:

Given:

Weight of the man, W = 60 N

We know that acceleration due to **gravity** at **height 'h'** from the **Earth's surface** is:

g' = $\frac{g}{\left[1+\left(\frac{h}{Re}\right)\right]^2}$

Where, **g = Acceleration due to gravity on the Earth's surface**

And, **R_e = Radius of the Earth**

For h = $\frac{R_e}{2}$

g' = $\frac{g}{\left[1+\left(\frac{Re}{2Re}\right)\right]^2}$

$\Rightarrow$ **g' =** $\frac{g}{\left[1+\left(\frac{1}{2}\right)\right]^2} = \frac{4}{9}\,g$

Also , the **weight of a body** of mass **'m' kg** at a height of **'h' meters** can be represented as :

W' = mg

= m × $\frac{4}{9}$g = $\frac{4}{9}$mg

= $\frac{4}{9}$W

= $\frac{4}{9}$ × 60 **= 26.66 N.**

Q.16: Considering the earth to be a sphere of uniform mass density. Find the weight of a body at one-third of the way down to the center of the earth if it weighed 200 N on the surface?

Sol:

Given,

The weight of a body having **mass 'm'** at the surface of earth, **W =** mg = **200 N**
Body of **mass 'm'** is located at **depth**, **d =** $\frac{1}{3}$ **R_e**
Where, **R_e = Radius of the Earth**
Now, acceleration due to **gravity g'** at **depth (d)** is given by the relation:
g' = $1-\left(\frac{d}{R_e}\right)g$

$\Rightarrow g' = 1-\left(\frac{R_e}{3R_e}\right)g = \frac{2}{3}\,g$

Now, weight of the body at depth d,
W' = mg'
W' = m × $\frac{2}{3}$g = $\frac{2}{3}$ mg

W' = $\frac{2}{3}$ W

⇒ W = $\frac{2}{3}$ × 200 **= 133.33 N**

Therefore, the weight of a body at one-third of the way down to the center of the earth = 133.33 N

Q.17: A missile is fired vertically upwards from the surface at 4 km m/s. What is the greatest height the missile can attain before falling back to the earth? Also find the final distance of the missile from the center of the earth. [Mass of earth = 6 x 10^{24} kg, mean radius of earth = 6.4 x 10^6m and take G =6.67 x 10^{-11} N m^2 kg^{-2}]

Sol:

Given:
Velocity of the **missile**, **v =** 4 km/s **= 4 × 10^3 m/s**
Mass of the Earth, M_E = 6 × 10^{24} kg
Radius of the **Earth**, **R_E = 6.4 × 10^6 m**

Let, the **height** reached by the **missile** be **'h'** and the mass of the **missile be** 'm**'.**
Now, at the **surface of the Earth**:
Total energy of the rocket at the surface of the Earth = Kinetic energy + Potential energy
T_{E1} = $\frac{1}{2}mv^2 + \frac{-GM_E\,m}{R_E}$

Now, at **highest point 'h':**

Kinetic Energy = 0 [Since, v = 0]
And, Potential energy = $\frac{-GM_E\,m}{R_E+h}$

Therefore, total energy of the missile at highest point 'h':

T_{E2} = 0 + $\frac{-GM_E\,m}{R_E+h}$

⇒ **T_{E2} =** $\frac{-GM_E\,m}{R_E+h}$

According to the **law of conservation of energy**, we have :
Total energy of the rocket at the **Earth's surface T_{E1}** = **Total energy** at **height 'h' T_{E2}:**

$\Rightarrow \frac{1}{2}mv^2 + \frac{-GM_E\,m}{R_E} = \frac{-GM_E\,m}{R_E+h}$

$\Rightarrow \frac{1}{2}v^2 + \frac{-GM_E}{R_E} = \frac{-GM_E}{R_E+h}$

$\Rightarrow v^2 = 2GM_E \times \left[\frac{1}{R_E} - \frac{1}{(R_E+h)}\right] = 2GM_E\left[\frac{h}{(R_E+h)R_E}\right]$

$\Rightarrow v^2 = \frac{g\,R_E\,h}{R_E+h}$

Where, $g = \frac{GM}{R_E^2}$ **= 9.8 ms^{-2}**

Therefore, v^2 (R_E + h) = 2gR_Eh

Therefore, $v^2(R_E + h) = 2gR_E h$

$\Rightarrow v^2 R_E = h\,(2gR_E - v^2)$

$$\Rightarrow h = \frac{R_E\, v^2}{2gR_E - v^2} = \frac{(6.4\times10^6)\times(4\times10^3)^2}{2\times9.8\times6.4\times10^6-(4\times10^3)^2}$$

$\Rightarrow$ **$h = 9.356 \times 10^5$ m**

Therefore, the **missile** reaches at height of 9.36×10^5 m from the surface.

Now, the **distance** from the **center** of the **earth = $h + R_E$** $= 9.36 \times 10^5 + 6.4 \times 10^6$ **$= 7.33 \times 10^6$ m**

Therefore, the greatest height the missile can attain before falling back to the earth is 9.356×10^5 m and the final distance of the missile from the center of the earth is 7.33×10^6 m

Q.18: On the surface of the earth escape velocity is 11.2 km/s. If a body (rocket) is projected upward with twice this speed, find the speed of this rocket at a distance far away from the surface of the earth. Neglect the presence of other heavenly bodies like the sun and other planets.

Sol:

Given,

Escape velocity of the Earth's surface, **v_{esc} = 11.2 km/s**
Projection velocity of the rocket, **$v_P = 2v_{esp}$**

Let, **mass** of the body **= m kg**

And, the **velocity** of the **rocket** at a **distance** very **far away** from the surface of earth **= v_F**

Total energy of the rocket on the surface **= $\frac{1}{2} m v_p^2 - \frac{1}{2} m v^2_{esp}$**

Total energy of the rocket at a distance very far away from the Earth **= $\frac{1}{2} m v_F^2$**

Now, **according** to the **law of conservation of energy**, we have:
$\frac{1}{2} m v_P^2 - \frac{1}{2} m v_{ESC}^2 = \frac{1}{2} m v_F^2$

$m v_{ESC}^2 = \frac{1}{2} m v_F^2$
$v_F = \sqrt{(v_p)^2 - (v_{esc})^2}$ **[Since, $v_P = 2v_{esp}$]**

$v_F = \sqrt{(2v_{esp})^2 - (v_{esc})^2}$

$v_F = \sqrt{3}\, v_{esc}$

$\Rightarrow v_F = \sqrt{3} \times 11.2$ **= 19.39 km/s**

Therefore, the speed of rocket at a distance far away from the surface of earth is 19.39 km/s.

Q.19: International Space Station (ISS) orbits the earth at a height of 380 km and has a mass of 420000 kg. Find the amount of energy required to take it out from the gravitational influence of earth. [Mass of earth = 6.0×10^{24} kg, radius of earth = 6.4×10^{6} m and take G = 6.67×10^{-11} N m^2 kg^{-2}]

Sol:

Given:

Mass of the **satellite, m = 420000 kg**

Radius of the **Earth, R_E = 6.4×10^6 m**

Mass of the **Earth, M_E = 6.0×10^{24} kg**
Universal gravitational constant, **G = 6.67×10^{-11} Nm^2 kg^{-2}**

Height of the **satellite**, **h** = 380 km **= 0.38×10^6 m**
We know that the **Total energy** of the satellite at **height 'h'** $= \frac{1}{2} m v^2 + \left[\frac{-GM_E\, m}{R_E + h}\right]$

Also, **orbital velocity** of the satellite, **v** $= \left[\frac{GM_E}{R_E+h}\right]^{\frac{1}{2}}$

Total energy at **height, h**$= \frac{1}{2}\left[\frac{GM_E}{R_E+h}\right] - \left[\frac{GM_E\, m}{R_E+h}\right]$

Therefore, T_E $= \frac{-1}{2}\left[\frac{GM_E\, m}{R_E+h}\right]$
This negative sign means that the ISS is bounded to Earth. This is the **bound energy** of the satellite.
Now, **Total Energy** required to send the satellite out of its orbit = **– (Bound energy)**
$T_E = \frac{1}{2}\left[\frac{GM_E\, m}{\ldots}\right]$

$T_E = \frac{1}{2}\left[\frac{(6.67\times10^{-11})\times(6\times10^{24})\times(4.2\times10^{5})}{(6.4\times10^{6})+(0.380\times10^{6})}\right] = \frac{1}{2}\times\left[\frac{1.681\times10^{20}}{6.78\times10^{6}}\right]$

$\Rightarrow$ **T_E = 1.2396 x 10^{13} J**

Therefore, **the amount of energy required to take ISS out from the gravitational influence of earth is 1.2396 x 10^{13} J**

Q.20: Two planets each of mass 2 x 10^{31} kg are on a path of collision with each other. If they have negligible speeds at a distance of 10^{10} km. Find the speed at which they collide if each planet has a radius of 10^3 km. [Take G = 6.67 x 10^{-11}]

Sol:

Given:
Radius of each planet, **R** = 10^3 km **= 10^6 m**

Distance between the planet, **r** = 10^{10} km **= 10^{13} m**

Mass of each planet, **M = 2 × 10^{31} kg**

For negligible speeds, **v = 0**

So the **total energy** of two planets separated by a **distance 'r'**:

$T_E = \frac{-GMM}{r} + \frac{1}{2}mv^2$

Since, v = 0;
Therefore, $T_E = \frac{-GMM}{r}$ . . . . **(1)**
Now, when the planets are just about to collide:

Let, the **velocity** of the planets **= v**

The centers of the two planets are at a **distance** of **= 2R**
Total kinetic energy of the two planets = $\frac{1}{2}Mv^2 + \frac{1}{2}Mv^2$ = **Mv^2**
Total potential energy of the two planets = $\frac{-GMM}{2R}$
Therefore, **Total energy** of the two stars = **$Mv^2 - \frac{GMM}{2R}$** . . . . **(2)**

Now, according to **the law of conservation of energy**:
$Mv^2 - \frac{GMM}{2R} = \frac{-GMM}{r}$

$v^2 = \frac{-GM}{r} + \frac{GM}{2R}$

$v^2 = GM \times \left[\frac{-1}{r} + \frac{1}{2R}\right]$

$\Rightarrow v^2 = 6.67 \times 10^{-11} \times 2 \times 10^{31} \times \left[\frac{-1}{(10^{13})} + \frac{1}{(2\times10^{6})}\right]$
$\Rightarrow v^2 = 6.67 \times 10^{14}$
Therefore, v = $(6.67 \times 10^{14})^{1/2}$ = 2.583 × 10^7 m/s.

Hence, the speed at which they will collide = 2.583 × 10^7 m/s.

Q.21: Two spheres each having a radius of 0.15 m and mass 1000 kg are placed 1.0 m apart from each other on a horizontal plane. Calculate the gravitational potential and force at the midpoint of the line connecting the centers of the two spheres. If an object is kept at that point will it be in equilibrium? If yes, is this equilibrium position is unstable or stable?

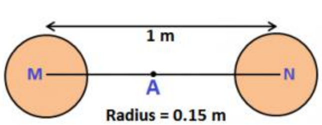
Sol:

Given:

Radius of spheres, **R = 0.15 m**

Distance between two spheres, **r = 1.0 m**

Mass of each sphere, **M = 1000 kg**



Two spheres

From the above figure, 'A' is the mid-point and since each sphere will exert the gravitational force in opposite direction. Therefore, **the gravitational force at this point will be zero.**

Gravitational potential at the midpoint (A) is;

U= $\left[\frac{-GM}{\frac{r}{2}} + \frac{-GM}{\frac{r}{2}}\right]$

U= $\left[\frac{-4GM}{r}\right]$

U= $\left[\frac{-4\times(6.67\times10^{-11})\times(1000)}{1.0}\right]$

$\Rightarrow$ **U= -2.668 x 10^{-7} J /kg**

Therefore, the gravitational potential and force at the mid-point of the line connecting the centers of the two spheres is = -2.668 x 10^{-7} J /kg

The net force on an object, placed at the mid-point is **zero**. However, if the object is displaced even a little towards any of the two bodies it will not return to its equilibrium position. **Thus, the body is in unstable equilibrium.**

Q.22: Find the gravitational potential due to Earth's gravity on a geo-stationary satellite orbiting earth at 36000 km above the surface. [Mass of earth = 6 x 10^{24} kg, radius = 6400 km and Assume that the potential energy is zero at infinity]

Sol:

Given:

Radius of the Earth, **R =** 6400 km **= 0.64 × 10^7 m**

Mass of Earth, **M = 6 x 10^{24} kg**

Height of the geo-stationary satellite from earth's surface, **h =** 36000 km = **3.6 x 10^7 m**

Therefore, **gravitational potential at height 'h'** on the geo-stationary satellite due to the earth's gravity:

$G_P = \frac{-GM}{R+h}$

$\Rightarrow G_P = \frac{-(6.67\times10^{-11})\times(6\times10^{24})}{(0.64\times10^7)+(3.6\times10^7)}$

$\Rightarrow G_P = \frac{-40.02\times10^{13}}{4.24\times10^7}$ **= -9.439 × 10^6 J/Kg**

Therefore, the gravitational potential due to Earth's gravity on a geo-stationary satellite orbiting earth is -9.439 × 10^6 J/Kg

Q.23: A star 10 times the size of the sun collapses on itself and gets reduced to a black hole of radius 10km, rotating at a speed of 10 revolutions per second. Prove that an object placed on its equator will remain stuck on the equator due to its gravity. [Mass of the sun = 2 x 10^3 kg]

Sol.

Any matter/ object will remain stuck to the surface if the **outward centrifugal force** is **lesser** than the **inward gravitational pull**.

Gravitational force, f_G = $\frac{GM\,m}{R^2}$ [Neglecting negative sign]

Here,
M = Mass of the star = 10 × 2 × 10^{30} = **2 × 10^{31} kg**
m = Mass of the object
R = Radius of the star = 10 km = **1 ×10^4 m**
Therefore, **f_G =** $\frac{(6.67\times10^{-11})\times(2\times10^{31})\,m}{(1\times10^4)^2} = 1.334\times10^{13} m\ N$

Now, **Centrifugal force, f_C = m r ω^2**
Here, ω = Angular speed = 2πv
v = Angular frequency = 10 rev s^{-1}
f_c = m R $(2\pi v)^2$

f_c = m × (10^4) × 4 × $(3.14)^2$ × $(10)^2$ **= (3.9 ×10^7m) N**

As f_G > f_C, the object will remain stuck to the surface of black hole

As $r_G > r_C$, the object will remain stuck to the surface of black hole.

Q.24: Find the amount of energy required to launch a spaceship stationed on Mars to out of the solar system.

(Mass of the Sun = 2 × 10^{30} kg; mass of the space ship = 2000 kg, mass of mars = 6.4 × 10^{23} kg; radius of mars = 3395 km; radius of the orbit of mars = 2.28 × 10^8 km; G= 6.67 × 10^{-11} m^2kg^{-2})

Sol:

Given,

Mass of the Sun, M = 2 × 10^{30} kg

Mass of the spaceship, **m_S = 2000 kg**

Radius of Mars, r = 3395 km **= 3.395 × 10^6 m**

Mass of Mars, M_m = 6.4 × 10 ^{23} kg

Orbital radius of Mars, R = 2.28 × 10^8 km = **2.28 × 10^{11} m**

Universal gravitational constant, G = 6.67 × 10^{-11} m^2 kg^{-2}

Now,

Potential energy of the spaceship due to the **gravity of Sun** = $\frac{-GMm_s}{R}$
Potential energy of the spaceship due to **gravity of Mars** = $\frac{-GM_M\ m_s}{r}$

As the spaceship is stationed on Mars, its **velocity** is **'zero'** and thus, its **kinetic energy is also 'zero'.**
Thus, **Total energy of the spaceship** $= \frac{-GM_M\ m_s}{r} - \frac{GMm_s}{R}$
$T_E = -Gm_s\left[\frac{M}{R} + \frac{M_M}{r}\right]$

The **negative sign** means that the system is in **bound state**.

Therefore, energy required to launch the spaceship out of the solar system:
T_E = – (bound energy)
$T_E = Gm_s\left[\frac{M}{R} + \frac{M_M}{r}\right]$

Now, on substituting the values of **G, M, R, m_s, M_M and r** we will get:

$T_E = (6.67 \times 10^{-11}) \times (2 \times 10^3) \times \left[\frac{2\times10^{30}}{2.28\times10^{11}} + \frac{6.4\times10^{23}}{3.395\times10^6}\right]$

$\Rightarrow\ 13.34 \times 10^{-8} \times [87.7 \times 10^{17}\ + 1.885 \times 10^{17}] = \mathbf{1.195 \times 10^{12} J}$

Therefore, the total amount of energy required to launch a spaceship stationed on Mars to out of the solar system = 1.195 × 10^{12} J

Q.25: A missile is shot vertically upward from the surface of Venus; if 25% of its initial energy is lost in overcoming Venus's atmospheric resistance, find the maximum height the missile will achieve before returning back to the surface. Take Initial velocity of the missile = 3 km s^{-1}, Mass of Venus = 4.8 × 10^{24} kg and the radius of Venus= 6052 km. [G = 6.67× 10^{-11} N m^2 kg^{-2}]

Sol.

Given:

Mass of Venus, **M = 4.8 × 10^{24} kg**

Initial velocity of the missile, **v = 3 km/s = 3 × 10^3 m/s**

Radius of Venus, **R** = 6052 km **= 6.05 x 10^6 m**

Universal gravitational constant, G = 6.67× 10^{-11} N m^2 kg^{-2}

Let the **mass** of the **missile** = **m kg**

Since, the Initial kinetic energy of the missile = $\frac{1}{2}$ mv^2
and the Initial potential energy of the missile = $\frac{-GMm}{R}$
Thus, total initial energy $= \frac{1}{2}mv^2 + \left(\frac{-GMm}{R}\right)$

It is given that **25 % of initial kinetic energy is lost** in overcoming the atmospheric resistance of Venus; this means that only **75% of the total kinetic energy** is **available** for **propelling** it upwards.

Hence, the total available initial energy $= \left[\frac{75}{100} \times \frac{1}{2}mv^2\right] - \frac{GMm}{R} = 0.375mv^2 - \frac{GMm}{R}$

Let **'h' be the maximum height attained by the missile.**
Now, at **height 'h'** the **final velocity = 0** and hence, the **kinetic energy = 0**

Therefore, the **total energy of the missile** at height 'h' = $\frac{-GMm}{R+h}$
Now, according to the law of conservation of energy:

$$\Rightarrow \quad 0.375mv^2 - \frac{GM}{R} = \frac{-GMm}{R+h}$$

$$\Rightarrow \quad 0.375\ v^2 = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\Rightarrow 0.375v^2 = GM \times \left[\frac{h}{R\ (R+h)}\right]$$

$$\Rightarrow \quad \frac{R+h}{h} = \frac{GM}{0.375\ v^2R}$$

$$\Rightarrow \quad \frac{R}{h} = \frac{GM}{0.375\ v^2R} - 1$$

$$\Rightarrow \quad \frac{R}{h} = \frac{GM - 0.375\ v^2R}{0.375\ v^2R}$$

$$\Rightarrow \quad \frac{1}{h} = \frac{GM - 0.375\ v^2R}{0.375\ v^2R^2}$$

$$\Rightarrow \quad h = \frac{0.375\ v^2R^2}{GM - 0.375\ v^2R}$$

Now, on substituting the values of **G, M, v and R** we will get:

$$\Rightarrow h = \frac{0.375 \times (3 \times 10^3)^2 \times (6.05 \times 10^6)^2}{[(6.67 \times 10^{-11}) \times (4.8 \times 10^{24})] - [0.375 \times (3 \times 10^3)^2 \times 6.05 \times 10^6]}$$

$$\Rightarrow h = \frac{0.375 \times (3 \times 10^3)^2 \times (6.05 \times 10^6)^2}{[(6.67 \times 10^{-11}) \times (4.8 \times 10^{24})] - [0.375 \times (3 \times 10^3)^2 \times 6.05 \times 10^6]}$$

$$\Rightarrow \quad \frac{1.235 \times 10^{20}}{3.2016 \times 10^{14} - 2.042 \times 10^{13}} = \mathbf{412023.75\ m = 4.12\ km}$$

Therefore, the maximum height that is achieved by the missile before returning back to the surface = 4.12 km.