NCERT SOLUTIONS
CLASS-XII MATHS
CHAPTER-10
VECTOR ALGEBRA

Exercise – 10.1

Question 1: Graphically represent a 40 km displacement towards 30 º east of north.

Answer 1:

Vector OP represent a 40 km displacement towards 30º east of north.

Question 2: Categorize the following measures as vectors and scalars.

(a) 20 kg (b) 4 meters north – south (c) 80º

(d) 70 watt (e) 10^{-17} coulomb (f) 56 m / s^{-2}

Answer 2:

(a) In 20 kg, only magnitude is involved. So, it is a scalar quantity.

(b) In 4 meters north – south, both the direction and magnitude are involved. So, it is a vector quantity

(c) In 80º, only magnitude is involved. So, it is a scalar quantity.

(d) In 70 watt, only magnitude is involved. So, it is a scalar quantity.

(e) In 10^{-17} coulombs, only magnitude is involved. So, it is a scalar quantity

(f) In 56 m / s^{-2}, both the direction and magnitude are involved. So, it is a vector quantity

Question 3: Categorize the following quantities as vector and scalar.

(a) Time period (b) distance (c) force

(d) Velocity (e) work done

Answer 3:

(a) In time period, only magnitude is involved. So, it is a scalar quantity

(b) In distance, only magnitude is involved. So, it is a scalar quantity

(c) In force, both the direction and magnitude are involved. So, it is a vector quantity

(d) In velocity, both the direction and magnitude are involved So, it is a vector quantity

(e) In work done, only magnitude is involved. So, it is a scalar quantity

Question 4: In the following diagram, recognize the corresponding vectors

(a) Coinitial

(b) Equal

(c) Collinear but not equal

Answer 4:

(a) Coinitial vectors are those vectors which have same initial point So, $\vec{a}\ and\ \vec{d}$ vectors are coinitial

(b) Equal vectors are vectors which have same magnitude and direction. So, $\vec{b}\ and\ \vec{d}$ vectors are equal.

(c) Collinear but not equal are those vectors which are parallel but has different directions. So, $\vec{a}\ and\ \vec{c}$ vectors are collinear but not equal.

Question 5: Check whether the following statements are true or false.

(a) $\vec{b}\ and\ \overrightarrow{-b}$ vectors are collinear

(b) The magnitudes of the two collinear are always equal.

(c) Collinear vectors are the two vectors having same magnitude.

Answer 5:

(a). True because the two vectors are parallel .

(b) False because collinear vectors must be parallel (b). False because collinear vectors must be parallel.

(c). False.

Exercise 10.2

Question 1: For the following vectors, calculate the magnitude of the following.

$\overrightarrow{m} = \hat{i} + \hat{j} + \hat{k}; \quad \overrightarrow{n} = \hat{2i} - \hat{7j} - \hat{3k}; \quad \overrightarrow{o} = \frac{1}{\sqrt{3}}\,\hat{i} + \frac{1}{\sqrt{3}}\,\hat{j} - \frac{1}{\sqrt{3}}\,\hat{k}$

Answer 1:

Given, $\overrightarrow{m} = \hat{i} + \hat{j} + \hat{k}; \quad \overrightarrow{n} = \hat{2i} - \hat{7j} - \hat{3k}; \quad \overrightarrow{o} = \frac{1}{\sqrt{3}}\,\hat{i} + \frac{1}{\sqrt{3}}\,\hat{j} - \frac{1}{\sqrt{3}}\,\hat{k}$

$$\left|\overrightarrow{m}\right| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$
$$\left|\overrightarrow{n}\right| = \sqrt{(2)^2 + (-7)^2 + (-3)^2} = \sqrt{4 + 49 + 9} = \sqrt{62}$$
$$\left|\overrightarrow{o}\right| = \sqrt{(\tfrac{1}{\sqrt{3}})^2 + (\tfrac{1}{\sqrt{3}})^2 + (\tfrac{1}{\sqrt{3}})^2} = \sqrt{\tfrac{1}{3} + \tfrac{1}{3} + \tfrac{1}{3}} = 1$$

Question-2: Mention two dissimilar vectors having similar magnitude?

Answer-2:

$$\overrightarrow{m} = (\hat{2i} - \hat{2j} + \hat{3k});\ and\ \overrightarrow{n} = (\hat{2i} + \hat{2j} - \hat{3k})$$
$$It\ can\ be\ observed\ that:$$
$$\left|\overrightarrow{m}\right| = \sqrt{(2)^2 + (-2)^2 + (3)^2} = \sqrt{17}\ and$$
$$\left|\overrightarrow{n}\right| = \sqrt{(2)^2 + (2)^2 + (3)^2} = \sqrt{17}$$

Thus, the two dissimilar vectors $\overrightarrow{m}\ and\ \overrightarrow{n}$ having the similar magnitude. Because of different directions the two vectors are dissimilar.

Question 3: Mention two dissimilar vectors having similar direction?

Answer 3:

Consider, $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}; \quad and\ \overrightarrow{b} = \hat{2i} + \hat{2j} + \hat{2k}$

$$The\ direction\ cosines\ of\ \overrightarrow{a}\ are\ given\ by,$$
$$p = \frac{1}{\sqrt{(1)^2+(1)^2+(1)^2}} = \frac{1}{\sqrt{3}},\ q = \frac{1}{\sqrt{(1)^2+(1)^2+(1)^2}} = \frac{1}{\sqrt{3}},\ and\ r = \frac{1}{\sqrt{(1)^2+(1)^2+(1)^2}} = \frac{1}{\sqrt{3}}$$

$$The\ direction\ cosines\ of\ \overrightarrow{b}\ are\ given\ by,$$
$$p = \frac{2}{\sqrt{(2)^2+(2)^2+(2)^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}},\ q = \frac{2}{\sqrt{(2)^2+(2)^2+(2)^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}\ and\ r = \frac{2}{\sqrt{(2)^2+(2)^2+(2)^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$The\ direction\ cosines\ of\ \overrightarrow{a}\ and\ \overrightarrow{b}\ are\ similar.$$

Thus, the direction of the two vectors is similar.

Question 4: $4\hat{i} + 5\hat{j}\ and\ p\hat{i} + q\hat{j}$ **are the vectors and they are equal. Obtain the values of p and q**

Answer 4:

Given, $4\hat{i} + 5\hat{j}\ and\ p\hat{i} + q\hat{j}$ are equal.

The equivalent components are equal.

So, the value of p = 4 and q = 5.

Question 5: The initial point of the vector is (3, 2) and the terminal point of the vector is (- 6, 8). Obtain the vector and scalar components of the given vector.

Answer 5:

Given,

The initial point of the vector A (3, 2) and the terminal point of the vector is B (- 6, 8).

The vector $\overrightarrow{AB} = (-6 - 3)\hat{i} + (8 - 2)\hat{j}$

$$\overrightarrow{AB} = -9\hat{i} + 6\hat{j}$$

The vector components of the given vector are $-9i$ and $6j$.

The scalar components of the given vector are – 9 and 6.

Question 6: The vectors $\overrightarrow{m} = \hat{i} + \hat{3j} + \hat{k}, \quad \overrightarrow{n} = -\hat{2i}-\hat{5j}-\hat{3k}, \ and\ \overrightarrow{o} = \hat{8i}-\hat{j}-\hat{2k}$. Obtain the sum.

Answer 6:

Given:

$$\overrightarrow{m} = \hat{i} + \hat{3j} + \hat{k}, \quad \overrightarrow{n} = -\hat{2i}-\hat{5j}-\hat{3k}, \ and\ \overrightarrow{o} = \hat{8i}-\hat{j}-\hat{2k}$$
$$\overrightarrow{m} + \overrightarrow{n} + \overrightarrow{o} = (1-2+8)\hat{i} + (3-5-1)\hat{j} + (1-3-2)\hat{k}$$
$$= 7\hat{i}-3\hat{j}-4\hat{k}$$

Question 7: Obtain the unit vector of $\overrightarrow{p} = \hat{i} + \hat{2j} + \hat{k}$ in the direction of the given vector.

Answer 7:

The unit vector $\hat{p}$ in the direction of vector $\overrightarrow{p} = \hat{i} + \hat{2j} + \hat{k}$

$$\left|\overrightarrow{p}\right| = \sqrt{(1)^2 + (2)^2 + (1)^2} = \sqrt{1+4+1} = \sqrt{6}$$
$$\hat{p} = \frac{\overrightarrow{p}}{\left|\overrightarrow{p}\right|} = \frac{\hat{i}+\hat{2j}+\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{2j} + \frac{1}{\sqrt{6}}\hat{k}$$
$$= \frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$$

Question 8: For a vector $\overrightarrow{AB}$, obtain the unit vector where the point A (2, 3, 4) and point B (5, 6, 7). The unit vector should be in the direction of given vector.

Answer 8:

Given, points A (2, 3, 4) and B (5, 6, 7).

$$\overrightarrow{AB} = (5-2)\hat{i} + (6-3)\hat{j} + (7-4)\hat{k}$$
$$\overrightarrow{AB} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$
$$\left|\overrightarrow{AB}\right| = \sqrt{(3)^2 + (3)^2 + (3)^2} = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$
$$The\ unit\ vector\ in\ the\ direction\ of\ \overrightarrow{AB}\ is$$
$$\frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = \frac{3\hat{i}+3\hat{j}+3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Question 9: In the direction of $\overrightarrow{m} + \overrightarrow{n}$, obtain the unit vector for given vectors $\overrightarrow{m} = \hat{3i}-\hat{j} + \hat{2k}\ and\ \overrightarrow{n} = \hat{2i}-\hat{3j}-\hat{k}$.

Answer 9:

Given,

The vectors $\overrightarrow{m} = \hat{3i}-\hat{j} + \hat{2k}\ and\ \overrightarrow{n} = \hat{2i}-\hat{3j}-\hat{k}$

$$\overrightarrow{m} = \hat{3i}-\hat{j} + \hat{2k}$$
$$\overrightarrow{n} = \hat{2i}-\hat{3j}-\hat{k}$$
$$\overrightarrow{m} + \overrightarrow{n} = (3+2)\hat{i} + (-1-3)\hat{j} + (2-1)\hat{k}$$
$$= 5\hat{i}-4\hat{j} + 1\hat{k}$$
$$\left|\overrightarrow{m} + \overrightarrow{n}\right| = \sqrt{(5)^2 + (-4)^2 + (1)^2} = \sqrt{25+16+1} = \sqrt{42}$$

Thus, in the direction of $\overrightarrow{m} + \overrightarrow{n}$, the vector is,

$$\frac{(\overrightarrow{m}+\overrightarrow{n})}{\left|\overrightarrow{m}+\overrightarrow{n}\right|} = \frac{5\hat{i}-4\hat{j}+1\hat{k}}{\sqrt{42}}$$
$$= \frac{1}{\sqrt{42}}5\hat{i}-\frac{1}{\sqrt{42}}4\hat{j} + \frac{1}{\sqrt{42}}\hat{k}$$
$$= \frac{5}{\sqrt{42}}\hat{i}-\frac{4}{\sqrt{42}}\hat{j} + \frac{1}{\sqrt{42}}\hat{k}$$

Question 10: A vector $6\hat{i}-2\hat{j} + 3\hat{k}$ has a magnitude of 8 units. Find the vector in the direction of given vector.

Answer 10:

Suppose, $\overrightarrow{m} = 6\hat{i}-2\hat{j} + 3\hat{k}$

$$\left|\overrightarrow{m}\right| = \sqrt{6^2 + (-2)^2 + 3^2} = \sqrt{36+4+9} = \sqrt{49} = 7$$
$$\hat{m} = \frac{\overrightarrow{m}}{\left|\overrightarrow{m}\right|} = \frac{6\hat{i}-2\hat{j}+3\hat{k}}{7}$$

$|m|$

Thus, the vector in the direction of given vector which has 8 units magnitude is given by,

$8\hat{m} = 8(\frac{6\hat{i}-2\hat{j}+3\hat{k}}{7}) = \frac{48}{7}\hat{i} - \frac{16}{7}\hat{j} + \frac{24}{7}\hat{k}$

Question 11: Prove whether the vectors $3\hat{i}-4\hat{j}+5\hat{k}$ and $9\hat{i}-12\hat{j}+15\hat{k}$ are collinear.

Answer 11:

Suppose, $\vec{p} = 3\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{q} = 9\hat{i}-12\hat{j}+15\hat{k}$

The condition for the vectors to be collinear is,

$\vec{q} = \lambda\vec{p}$

Accordingly,

$9\hat{i}-12\hat{j}+15\hat{k} = 3\ (3\hat{i}-4\hat{j}+5\hat{k})$, which satisfies the condition with $\lambda = 3$

Hence, proved

Question 12: Obtain the direction cosines of the vectors $2\hat{i}-4\hat{j}+6\hat{k}$

Answer 12:

$\vec{m} = 2\hat{i}-4\hat{j}+6\hat{k}$
$\left|\vec{m}\right| = \sqrt{(2)^2+(-4)^2+(6)^2} = \sqrt{4+16+36} = \sqrt{56}$
$Thus,\ the\ direction\ cosines\ of\ \vec{m}\ are\ \left(\frac{2}{\sqrt{56}}, \frac{-4}{\sqrt{56}}, \frac{6}{\sqrt{56}}\right)$

Question 13: P (1, 2, – 3) and Q (- 1, – 2, 1) are the joining points of a vector directed from P to Q. Obtain the direction cosines of the vector.

Answer 13:

P (1, 2, – 3) and Q (- 1, – 2, 1) are the joining points of a vector.

$\overrightarrow{PQ} = (-1-1)\hat{a} + (-2-2)\hat{b} + (1-(-3))\hat{c}$
$\overrightarrow{PQ} = (-2)\hat{a} + (-4)\hat{b} + (4)\hat{c}$
$\left|\overrightarrow{PQ}\right| = \sqrt{(-2)^2+(-4)^2+4^2} = \sqrt{4+16+16} = \sqrt{36} = 6$

The direction cosines of the vector $\overrightarrow{PQ}$ are $\left(-\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right) = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$

Question 14: Prove that the $\hat{i}+\hat{j}+\hat{k}$ is evenly tending to the axes OX, OY and OZ

Answer 14:

Suppose, $\vec{m} = \hat{i}+\hat{j}+\hat{k}$
$Then,\ \left|\vec{m}\right| = \sqrt{(1)^2+(1)^2+(1)^2} = \sqrt{3}$

The direction cosines of the vector $\vec{m}$ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Now, let α, β, and γ be the angles formed by with the positive directions of x, y, and z axes.

Thus, we obtain,

$cos\ \alpha = \frac{1}{\sqrt{3}},\ cos\ \beta = \frac{1}{\sqrt{3}}\ and\ cos\ \gamma = \frac{1}{\sqrt{3}}$

Thus, the vector is evenly tending to the axes OX, OY and OZ

Question 15: The position vectors of a joining points A and B are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively. Obtain the position vector of C which divides the given points in 2 : 1 ratio.

(a) Internally

(b) Externally

Answer 15:

The position vector of C which divides the given points in m : n ratio is written as:

(a) Internally: $\frac{m\vec{b}+n\vec{a}}{m+n}$

(b) Externally: $\frac{m\vec{b}-n\vec{a}}{m-n}$

Given,

Position vectors of a joining points A and B,

$OA = \imath + 2\jmath - k$ and $OB = \imath + \jmath + k$

(a) The position vector of point C which divides the line joining two points A and B internally in the ratio 2:1 is given by,

$$\overrightarrow{OC} = \frac{2(-\hat{i}+\hat{j}+\hat{k})+1(\hat{i}+2\hat{j}-\hat{k})}{2+1} = \frac{(-2\hat{i}+2\hat{j}+2\hat{k})+(\hat{i}+2\hat{j}-\hat{k})}{3}$$
$$= \frac{-\hat{i}+4\hat{j}+\hat{k}}{3}$$
$$=-\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$$

(b) The position vector of point C which divides the line joining two points A and B externally in the ratio 2:1 is given by,

$$\overrightarrow{OC} = \frac{2(-\hat{i}+\hat{j}+\hat{k})-1(\hat{i}+2\hat{j}-\hat{k})}{2-1}$$
$$= (-2\hat{i}+2\hat{j}+2\hat{k})+(\hat{i}+2\hat{j}-\hat{k})$$
$$=-3\hat{i}+3\hat{k}$$

Question 16: A (3, 4, 5) and B (5, 2, – 3) are the joining points of a vector. Obtain the midpoint position vector.

Answer 16:

The midpoint position vector with the joining points A (3, 4, 5) and B (5, 2, – 3),

Suppose, $\overrightarrow{OC}$ be the required vector, then,

$$\overrightarrow{OC} = \frac{(3\hat{i}+4\hat{j}+5\hat{k})+(5\hat{i}+2\hat{j}+(-3)\hat{k})}{2} = \frac{(3+5)\hat{i}+(4+2)\hat{j}+(5-3)\hat{k}}{2}$$
$$= \frac{8\hat{i}+6\hat{j}+2\hat{k}}{2}$$
$$= 4\hat{i}+3\hat{j}+\hat{k})$$

Question 17: Prove that the points P, Q and R with position vectors, $\overrightarrow{p} = 3\hat{a}-4\hat{b}-4\hat{c},\ \overrightarrow{q} = 2\hat{a}-\hat{b}+\hat{c},\ and\ \overrightarrow{r} = \hat{a}-3\hat{b}-5\hat{c}$ respectively from the vertices of a right angled triangle.

Answer 17:

Given,

The points P, Q and R with position vectors $\overrightarrow{p} = 3\hat{a}-4\hat{b}-4\hat{c},\ \overrightarrow{q} = 2\hat{a}-\hat{b}+\hat{c},\ and\ \overrightarrow{r} = \hat{a}-3\hat{b}-5\hat{c}$

$\overrightarrow{p} = 3\hat{a}-4\hat{b}-4\hat{c},\ \overrightarrow{q} = 2\hat{a}-\hat{b}+\hat{c},\ and\ \overrightarrow{r} = \hat{a}-3\hat{b}-5\hat{c}$ $\overrightarrow{PQ} = \overrightarrow{q}-\overrightarrow{p} = (2-3)\hat{a}+(-1+4)\hat{b}+(1+4)\hat{c} =-\hat{a}+3\hat{b}+5\hat{c}$

$$\overrightarrow{QR} = \overrightarrow{r}-\overrightarrow{q} = (1-2)\hat{a}+(-3+1)\hat{b}+(-5-1)\hat{c} =-\hat{a}-2\hat{b}-6\hat{c}$$
$$\overrightarrow{RP} = \overrightarrow{p}-\overrightarrow{r} = (3-1)\hat{a}+(-4+3)\hat{b}+(-4+5)\hat{c} = 2\hat{a}-\hat{b}+\hat{c}$$

$$\left|\overrightarrow{PQ}\right|^2 = (-1)^2+3^2+5^2 = 1+9+25 = 35$$
$$\left|\overrightarrow{QR}\right|^2 = (-1)^2+(-2)^2+(-6)^2 = 1+4+36 = 41$$
$$\left|\overrightarrow{RP}\right|^2 = 2^2+(-1)^2+1^2 = 4+1+1 = 6$$
$$\left|\overrightarrow{PQ}\right|^2+\left|\overrightarrow{QR}\right|^2 = 35+6 = 41 = \left|\overrightarrow{QR}\right|^2$$

Hence, PQR is a right angled triangle.

Question 18: Which of the following is incorrect in the triangle PQR?

$$(i)\overrightarrow{PQ}+\overrightarrow{QR}+\overrightarrow{RP} = 0$$
$$(ii)\overrightarrow{PQ}+\overrightarrow{QR}-\overrightarrow{PR} = 0$$
$$(iii)\overrightarrow{PQ}+\overrightarrow{QR}-\overrightarrow{RP} = 0$$
$$(iv)\overrightarrow{PQ}-\overrightarrow{RQ}+\overrightarrow{RP} = 0$$

Answer 18:

In a given triangle, applying the triangle law of addition, we get,

$$\overrightarrow{PQ}+\overrightarrow{QR} = \overrightarrow{PR}\ldots.(1)$$
$$\overrightarrow{PQ}+\overrightarrow{QR} =-\overrightarrow{RP}$$
$$\overrightarrow{PQ}+\overrightarrow{QR}+\overrightarrow{RP} = \overrightarrow{0}\ldots.(2)$$
$$Statement\ (i)\ is\ true$$
$$\overrightarrow{PQ}+\overrightarrow{QR} = \overrightarrow{PR}$$

$\acute{PQ} + \acute{QR} – \acute{PR} = \acute{0}$
$Statement\ (ii)\ is\ true$

From equation (2), we get:

$\overrightarrow{PQ} – \overrightarrow{RQ} + \overrightarrow{RP} = \overrightarrow{0}$
$Statement\ (iv)\ is\ true$
$Considering\ statement\ (iii)$
$\overrightarrow{PQ} + \overrightarrow{QR} – \overrightarrow{RP} = \overrightarrow{0}$
$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{RP} \ldots\ldots (3)$

From equations (3) and (1), we get:

$\overrightarrow{PR} = \overrightarrow{RP}$
$\overrightarrow{PR} = – \overrightarrow{PR}$
$2\overrightarrow{PR} = \overrightarrow{0}$
$\overrightarrow{PR} = \overrightarrow{0},\ is\ not\ true.\ Statement\ (iii)\ is\ true$

Question 19: Check whether the corresponding statements are true if the two vectors $\overrightarrow{p}\ and\ \overrightarrow{q}$ are collinear.

$(i)\ \overrightarrow{q} = \lambda \overrightarrow{p},\ for\ some\ scalar\ \lambda$
$(ii)\ \overrightarrow{p} = \pm \overrightarrow{q}$
$(iii) The\ components\ of\ \overrightarrow{p}\ and\ \overrightarrow{q}\ are\ proportional$
$(iv)\ \overrightarrow{p}\ and\ \overrightarrow{q}\ have\ different\ magnitudes\ and\ have\ similar\ direction.$

Answer 19:

The two vectors are said to be collinear when they are parallel to each other.

$\overrightarrow{p}\ and\ \overrightarrow{q}$ are collinear vectors.

Thus, we have,

$\overrightarrow{q} = \lambda \overrightarrow{p},(for\ some\ scalar\ \lambda)$
$Suppose,\ \lambda = \pm 1,\ then\ \overrightarrow{q} = \pm 1 \overrightarrow{p}$
$If,\ \overrightarrow{p} = p_1\hat{i} + p_2\hat{j} + p_3\hat{k},\ \overrightarrow{q} = q_1\hat{i} + q_2\hat{j} + q_3\hat{k},\ then\ \overrightarrow{q} = \lambda \overrightarrow{p}.$

$q_1\hat{i} + q_2\hat{j} + q_3\hat{k} = \lambda(p_1\hat{i} + p_2\hat{j} + p_3\hat{k})$
$q_1\hat{i} + q_2\hat{j} + q_3\hat{k} = (\lambda p_1)\hat{i} + (\lambda p_2)\hat{j} + (\lambda p_3)\hat{k}$
$q_1 = \lambda p_1, q_2 = \lambda p_2, q_3 = \lambda p_3$
$=> \frac{q_1}{p_1} = \frac{q_2}{p_2} = \frac{q_3}{p_3} = \lambda$

Hence, the components of $\overrightarrow{p}\ and\ \overrightarrow{q}$ are proportional.

Though, vectors $\overrightarrow{p}\ and\ \overrightarrow{q}$ can have different directions.

Thus, statement (iv) is incorrect.

Exercise 10.3

Q.1 : Find the angle between two vectors $\vec{m}$ and $\vec{n}$ with magnitude $\sqrt{3}$and 2 , respectively having $\vec{m}.\vec{n} = \sqrt{6}$

Solution 1:

It is given that,

$|\vec{m}| = \sqrt{3}$,

$|\vec{n}|$ = 2

And $\vec{m}.\vec{n} = \sqrt{6}$

$\vec{m}.\vec{n} = |\vec{m}|\,|\vec{n}| \cos\theta$

Now, we know that

Therefore,

$\Rightarrow \sqrt{6} = \sqrt{3} \times 2 \times cos\theta$

$\Rightarrow \cos\theta = \frac{\sqrt{6}}{\sqrt{3}\times 2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}$

Therefore, the angle between the given vectors $\vec{m}$ and $\vec{n}$ is $\frac{\pi}{4}$

Q 2 : Find the angle between the vectors $\hat{a} – 2\hat{b} + 3\hat{c} and 3\hat{a} – 2\hat{b} + \hat{c}$

Solution 2:

The given vectors are:

$\vec{m} = \hat{a} - 2\hat{b} + 3\hat{c} and \vec{n} = 3\hat{a} - 2\hat{b} + \hat{c}$ $|\vec{m}| = \sqrt{1^2 + (-2)^2 + 3^2}$ $|\vec{m}| = \sqrt{1+4+9}$ $|\vec{m}| = \sqrt{14}$ $|\vec{n}| = \sqrt{3^2 + (-2)^2 + 1^2}$ $|\vec{n}| = \sqrt{9+4+1}$ $|\vec{n}| = \sqrt{14}$
$Now, \vec{m}.\vec{n} = \left(\hat{a} - 2\hat{b} + 3\hat{c}\right)\left(3\hat{a} - 2\hat{b} + \hat{c}\right)$ $Now, \vec{m}.\vec{n} = 1.3 + (-2)(-2) + 3.1$ $Now, \vec{m}.\vec{n} = 3 + 4 + 3$ $Now, \vec{m}.\vec{n} = 10$

Also, we know that

$\vec{m}.\vec{n} = |\vec{m}|\,|\vec{n}|\cos\theta$

Therefore,

$10 = \sqrt{14}\sqrt{14}\cos\theta$ $\cos\theta = \frac{10}{14}$ $\theta = \cos^{-1}\frac{5}{7}$

Q 3. Find the projection of the vector $\hat{a} - \hat{b}$ on the vector $\hat{a} + \hat{b}$.

Solution 3:

Let, $\hat{i} = \hat{a} - \hat{b}$

And $\hat{j} = \hat{a} + \hat{b}$

Now, projection of vector $\vec{i}$ on $\vec{j}$ is given by,

$\frac{1}{|\vec{j}|}\left(\vec{i}.\vec{j}\right) = \frac{1}{\sqrt{1+1}}\{1.1 + (-1)(1)\} = \frac{1}{\sqrt{2}}(1-1) = 0$

Hence the projection of vector $\vec{i}$ on $\vec{j}$ is 0

Q 4. Find the projection of the vector $\hat{a} + 3\hat{b} + 7\hat{c}$ on the vector $7\hat{a} - \hat{b} + 8\hat{c}$

Solution 4:

Let $\hat{i} = \hat{a} + 3\hat{b} + 7\hat{c}$ and $\hat{j} = 7\hat{a} - \hat{b} + 8\hat{c}$

Now, projection of vector $\vec{i}$ on $\vec{j}$ is given by,

$\frac{1}{j}\left(\vec{i}.\vec{j}\right) = \frac{1}{\sqrt{7^2+(-1)^2+8^2}}\{1(7) + 3(-1) + 7(8)\}$ $\frac{1}{j}\left(\vec{i}.\vec{j}\right) = \frac{7-3+56}{\sqrt{49+1+64}}$ $\frac{1}{j}\left(\vec{i}.\vec{j}\right) = \frac{60}{\sqrt{114}}$

Q 5: Show that each of the given three vectors is a unit vector :

$\frac{1}{7}\left(2\hat{a} + 3\hat{b} + 6\hat{c}\right), \frac{1}{7}\left(3\hat{a} - 6\hat{b} + 2\hat{c}\right), \frac{1}{7}\left(6\hat{a} + 2\hat{b} - 3\hat{c}\right)$

Also, show that they are mutually perpendicular to each other.

Solution 5:

$Let \vec{i} = \frac{1}{7}\left(2\hat{a} + 3\hat{b} + 6\hat{c}\right) = \frac{2}{7}\hat{a} + \frac{3}{7}\hat{b} + \frac{6}{7}\hat{c}$
$Let \vec{j} = \frac{1}{7}\left(3\hat{a} - 6\hat{b} + 2\hat{c}\right) = \frac{3}{7}\hat{a} - \frac{6}{7}\hat{b} + \frac{2}{7}\hat{c}$
$Let \vec{k} = \frac{1}{7}\left(6\hat{a} + 2\hat{b} - 3\hat{c}\right) = \frac{6}{7}\hat{a} + \frac{2}{7}\hat{b} - \frac{3}{7}\hat{c}$

$\Rightarrow \left|\vec{i}\right| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$

$\left|\vec{i}\right| = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}}$ $\left|\vec{i}\right| = 1$

$\Rightarrow \left|\vec{j}\right| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(\frac{-6}{7}\right)^2 + \left(\frac{2}{7}\right)^2}$

$\left|\vec{j}\right| = \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}}$ $\left|\vec{j}\right| = 1$

$\Rightarrow \left|\vec{k}\right| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(\frac{-3}{7}\right)^2}$

$\left|\vec{k}\right| = \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}}$ $\left|\vec{k}\right| = 1$

Thus, each of the given three vectors is a unit vector.

$\Rightarrow \vec{i}.\vec{j} = \frac{2}{7} \times \frac{3}{7} + \frac{3}{7} \times \left(\frac{-6}{7}\right) + \frac{6}{7} \times \frac{2}{7}$

$\vec{i}.\vec{j} = \frac{6}{49} - \frac{18}{49} + \frac{12}{49}$ $\vec{i}.\vec{j} = 0$

$\Rightarrow \vec{j}.\vec{k} = \frac{3}{7} \times \frac{6}{7} + \frac{-6}{7} \times \left(\frac{2}{7}\right) + \frac{2}{7} \times \frac{-3}{7}$

$\vec{j}.\vec{k} = \frac{18}{49} - \frac{12}{49} - \frac{6}{49}\ \vec{j}.\vec{k} = 0$

$\Rightarrow \vec{k}.\vec{i} = \frac{6}{7} \times \frac{2}{7} + \frac{2}{7} \times \left(\frac{3}{7}\right) + \frac{-3}{7} \times \frac{6}{7}$

$\vec{k}.\vec{i} = \frac{12}{49} - \frac{6}{49} - \frac{18}{49}\ \vec{k}.\vec{i} = 0$

Hence, the given three vectors are mutually perpendicular to each other.

Q 6: Find:

$|\vec{m}|\, and\, |\vec{n}|$, **if** $(\vec{m}+\vec{n}).(\vec{m}+\vec{n}) = 8$ **and** $|\vec{m}| = 8\,|\vec{n}|$

Solution 6:

$(\vec{m}+\vec{n}).(\vec{m}-\vec{n}) = 8$

$\Rightarrow \vec{m}.\vec{m} - \vec{m}\vec{n} + \vec{n}\vec{m} - \vec{n}.\vec{n} = 8$

$\Rightarrow |\vec{m}|^2 - |\vec{n}|^2 = 8$

$\Rightarrow 8|\vec{n}|^2 - |n|^2 = 8 \,.........\, [|\vec{m} = 8\,|\vec{n}||]$

$\Rightarrow 64|\vec{n}|^2 - |\vec{n}|^2 = 8$

$\Rightarrow 63|\vec{n}|^2 = 8$

$\Rightarrow |\vec{n}|^2 = \frac{8}{63}$

$\Rightarrow |\vec{n}|^2 = \sqrt{\frac{8}{63}}$ [magnitude of a vector is non-negative]

$\Rightarrow |\vec{n}|^2 = \frac{2\sqrt{2}}{3\sqrt{7}}$

$|\vec{m}| = 8\,|\vec{n}|\ |\vec{m}| = \frac{8\times2\sqrt{2}}{3\sqrt{7}}\ |\vec{m}| = \frac{16\sqrt{2}}{3\sqrt{7}}$

Q 7: Find the product of the following : $(3\vec{m}-5\vec{n}).(2\vec{m}+7\vec{n})$

Solution 7:

$(3\vec{m}-5\vec{n}).(2\vec{m}+7\vec{n}) = 3\vec{m}.2\vec{m} + 3\vec{m}.7\vec{n} - 5\vec{n}.2\vec{m} - 5\vec{n}.7\vec{n} = 6\vec{m}.\vec{m} + 21\vec{m}.\vec{n} - 10\vec{n}.\vec{m} - 35\vec{n}.\vec{n} = 6|\vec{m}|^2 + 11\vec{m}.\vec{n} - 35|\vec{n}|^2$

Q 8: Find the magnitude of two vectors $\vec{m} and \vec{n}$**, having the same magnitude and such that the angle between them is 60° and their scalar product is** $\frac{1}{2}$

Solution 8:

Let θ be the angle between the vectors $\vec{m} and \vec{n}$.

As given in the question:

$|\vec{m}| = |\vec{n}|\,, \vec{m}.\vec{n} = \frac{1}{2} and \theta = 60^\circ \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ **(1)**

We know that:

$\vec{m}.\vec{n} = |\vec{m}|\,|\vec{n}|\cos\theta$

Therefore,

$\frac{1}{2} = |\vec{m}|\,|\vec{m}|\, cos60^\circ \ldots\ldots\ldots\ldots\ldots\ldots$ **[using (1)]**

$\Rightarrow \frac{1}{2} = |\vec{m}|^2 \times \frac{1}{2}$

$\Rightarrow |\vec{m}|^2 = 1$

$\Rightarrow |\vec{m}|^2 = |\vec{n}|^2 = 1$

Q 9: Find:

$|\vec{y}|$ **, if for a unit vector** $\vec{b}$ **,** $\left(\vec{y}-\vec{b}\right).\left(\vec{y}+\vec{b}\right) = 12$

Solution 9:

$\left(\vec{y}-\vec{b}\right).\left(\vec{y}+\vec{b}\right) = 12$

$\Rightarrow \vec{y}.\vec{y} + \vec{y}.\vec{b} - \vec{b}.\vec{y} - \vec{b}.\vec{b} = 12$

$\Rightarrow |\vec{y}|^2 - \left|\vec{b}\right|^2 = 12$

$\Rightarrow |\vec{y}|^2 - 1 = 12 \left[\left|\vec{b}\right| = 1 as \vec{b} is a unit vector\right]$

$\Rightarrow |\vec{y}|^2 = 13$

Therefore,

$|\vec{y}| = \sqrt{13}$

Q 10: If $\vec{i} = 2\hat{a} + 2\hat{b} + 3\hat{c}$,

$\vec{j} = -\hat{a} + 2\hat{b} + \hat{c}$

and $\vec{k} = 3\hat{a} + \hat{b}$ are such that $\vec{i} + \lambda\vec{j}$ is perpendicular to $\vec{k}$ then find the value of λ

Solution 10:

The given vectors are $\vec{i} = 2\hat{a} + 2\hat{b} + 3\hat{c}$, $\vec{j} = -\hat{a} + 2\hat{b} + \hat{c}$ and $\vec{k} = 3\hat{a} + \hat{b}$

Now,

$\vec{i} + \lambda\vec{j} = \left(2\hat{a} + 2\hat{b} + 3\hat{c}\right) + \lambda\left(-\hat{a} + 2\hat{b} + \hat{c}\right) = (2 - \lambda)\hat{a} + (2 + 2\lambda)\hat{b} + (3 + \lambda)\,\hat{c}$

If $\left(\vec{i} + \lambda\vec{j}\right)$ is perpendicular to $\vec{k}$,then

$\left(\vec{i} + \lambda\vec{j}\right).\vec{k} = 0$

$\Rightarrow \left[(2 - \lambda)\,\hat{a} + (2 + 2\lambda)\,\hat{b} + (3 + \lambda)\,\hat{c}\right]\left(3\hat{a} + \hat{b}\right) = 0$

$\Rightarrow [(2 - \lambda)\,3 + (2 + 2\lambda)\,1 + (3 + \lambda)\,0] = 0$

$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$

$\Rightarrow -\lambda + 8 = 0$

$\Rightarrow \lambda = 8$

Hence, the required value of λ is 8.

Q 11: Show that:

$|\vec{a}|\,\vec{b} + \left|\vec{b}\right|\vec{a}$ is perpendicular to $|\vec{a}|\,\vec{b} - \left|\vec{b}\right|\vec{a}$, for any two non zero vectors $\vec{a} and \vec{b}$

Solution 11:

$\Rightarrow \left(|\vec{a}|\,\vec{b} + \left|\vec{b}\right|\vec{a}\right).\left(|\vec{a}|\,\vec{b} - \left|\vec{b}\right|\vec{a}\right)$

$\left(|\vec{a}|\,\vec{b} + \left|\vec{b}\right|\vec{a}\right).\left(|\vec{a}|\,\vec{b} - \left|\vec{b}\right|\vec{a}\right)$ = $|\vec{a}|^2\vec{b}.\vec{b} - |\vec{a}|\left|\vec{b}\right|\vec{b}.\vec{a} + \left|\vec{b}\right||\vec{a}|\,\vec{a}.\vec{b} - \left|\vec{b}\right|^2\vec{a}.\vec{a}$

$= |\vec{a}|^2\left|\vec{b}\right|^2$

$\left(|\vec{a}|\,\vec{b} + \left|\vec{b}\right|\vec{a}\right).\left(|\vec{a}|\,\vec{b} - \left|\vec{b}\right|\vec{a}\right)$ = 0

Hence, $|\vec{a}|\,\vec{b} + \left|\vec{b}\right|\vec{a}$ is perpendicular to $|\vec{a}|\,\vec{b} - \left|\vec{b}\right|\vec{a}$

Q 12: If $\vec{a}.\vec{a} = 0 and \vec{a}.\vec{b} = 0$, then what can be concluded about the vector $\vec{b}$?

Solution:

It is given that $\vec{a}.\vec{a} = 0 and \vec{a}.\vec{b} = 0$

Now,

that $\vec{a}.\vec{a} = 0$

$\Rightarrow |\vec{a}|^2 = 0$

$\Rightarrow |\vec{a}| = 0$

Therefore, $\vec{a}$ is a zero vector

Hence, vector $\vec{b}$ satisfying $\vec{a}.\vec{b} = 0$ can be any vector

Q 13:

If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c}$ = 0, find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

Solution 13:

$|\vec{a} + \vec{b} + \vec{c}|^2 = \left(\vec{a} + \vec{b} + \vec{c}\right)\left(\vec{a} + \vec{b} + \vec{c}\right)$

$$\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = |\vec{a}|^2+\left|\vec{b}\right|^2+|\vec{c}|^2+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)$$

$$\Rightarrow 0 = 1+1+1+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)$$

$$\Rightarrow \left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right) = \frac{-3}{2}$$

Q 14: If either vector $\vec{a}=\vec{0} or \vec{b}=\vec{0}$, then $\vec{a}.\vec{b}=0$. But the converse need not be true. Justify your answer with an example.

Solution 14:

Consider $\vec{a}=2\hat{i}+4\hat{j}+3\hat{k} and \vec{b}=3\hat{i}+3\hat{j}-6\hat{k}$

Then,

$$\vec{a}.\vec{b}=2.3+4.3+3(-6)=6+12-18=0$$

We now observe that :

$$|\vec{a}|=\sqrt{2^2+4^2+3^2}=\sqrt{29}$$

Therefore,

$$\vec{a}\neq\vec{0}\ \left|\vec{b}\right|=\sqrt{3^2+3^2+(-6)^2}=\sqrt{54}$$

Therefore,

$$\vec{b}\neq\vec{0}$$

Hence, the converse of the given statement need not be true

Q 15: If the vertices A, B, C of a triangle ABC are (1 , 2 , 3) , (– 1 , 0 , 0) , (0 , 1 , 2) , respectively, then find ∠ ABC. [∠ ABC is the angle between the vectors $\overrightarrow{BA} and \overrightarrow{BC}$]

Solution 15:

The vertices of ΔABC are given as A (1 , 2 , 3) , B (– 1 , 0 , 0) , and C (0 , 1 , 2). Also, it is given that ∠ ABC is the angle between the vectors $\overrightarrow{BA} and \overrightarrow{BC}$

$$\overrightarrow{BA}=\{1-(-1)\}\hat{i}+(2-0)\hat{j}+(3-0)\hat{k}=2\hat{i}+2\hat{j}+3\hat{k}\ \overrightarrow{BC}=\{0-(-1)\}\hat{i}+(1-0)\hat{j}+(2-0)\hat{k}=\hat{i}+\hat{j}+2\hat{k}$$

Therefore,

$$\overrightarrow{BA}.\overrightarrow{BC}=\left(2\hat{i}+2\hat{j}+3\hat{k}\right).\left(\hat{i}+\hat{j}+2\hat{k}\right)=2\times1+2\times1+3\times2=2+2+6=10\ \left|\overrightarrow{BA}\right|=\sqrt{2^2+2^2+3^2}=\sqrt{4+4+9}=\sqrt{17}$$
$$\left|\overrightarrow{BC}\right|=\sqrt{1+1+2^2}=\sqrt{6}$$

Now, it is known that :

$$\overrightarrow{BA}.\overrightarrow{BC}=\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|cos\left(\angle ABC\right)$$

Therefore,

$$10=\sqrt{17}\times\sqrt{6}cos\left(\angle ABC\right)$$

$$\Rightarrow cos\left(\angle ABC\right)=\frac{10}{\sqrt{17}\times\sqrt{6}}$$

$$\Rightarrow \angle ABC=\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

Q 16: Show that the points A (1 , 2 , 7) , B (2 , 6 , 3) and C (3 , 10 , – 1) are collinear.

Solution 16:

The given points are A (1 , 2 , 7) , B (2 , 6 , 3) , and C (3 , 10 , – 1).

Therefore,

$$\overrightarrow{AB}=(2-1)\hat{i}+(6-2)\hat{j}+(3-7)\hat{k}=\hat{i}+4\hat{j}-4\hat{k}\ \overrightarrow{BC}=(3-2)\hat{i}+(10-6)\hat{j}+(-1-3)\hat{k}=\hat{i}+4\hat{j}-4\hat{k}$$
$$\overrightarrow{AC}=(3-1)\hat{i}+(10-2)\hat{j}+(-1-7)\hat{k}=2\hat{i}+8\hat{j}-8\hat{k}$$

$$\Rightarrow \left|\overrightarrow{AB}\right|=\sqrt{1^2+4^2+(-4)^2}=\sqrt{1+16+16}=\sqrt{33}$$

$$\Rightarrow \left|\overrightarrow{BC}\right|=\sqrt{1^2+4^2+(-4)^2}=\sqrt{1+16+16}=\sqrt{33}$$

$\Rightarrow \left|\overrightarrow{AC}\right| = \sqrt{2^2+8^2+8^2} = \sqrt{4+64+64} = \sqrt{132} = 2\sqrt{33}$

Therefore,

$\left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$

Hence, the given points A, B, and C are collinear.

Q 17: Show that the vectors $2\hat{i}+\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k} and 3\hat{i}-4\hat{j}-4\hat{k}$ form the vertices of a right angled triangle.

Solution 17:

Let vectors $2\hat{i}+\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k} and 3\hat{i}-4\hat{j}-4\hat{k}$ be position vectors of points A, B, and C respectively.

i.e , $\overrightarrow{OA} = 2\hat{i}-\hat{j}+\hat{k}, OB = \hat{i}-3\hat{j}-5\hat{k} and OC = 3\hat{i}-4\hat{j}-4\hat{k}$

Therefore,

$\overrightarrow{AB} = (1-2)\,\hat{i} + (-3+1)\,\hat{j} + (-5-1)\,\hat{k} = -\hat{i}-2\hat{j}-6\hat{k}\ \overrightarrow{BC} = (3-1)\,\hat{i} + (-4+3)\,\hat{j} + (-4+5)\,\hat{k} = -2\hat{i}-\hat{j}-\hat{k}$
$\overrightarrow{AC} = (2-3)\,\hat{i} + (-1+4)\,\hat{j} + (1+4)\,\hat{k} = -\hat{i}+3\hat{j}+5\hat{k}$

$\Rightarrow \left|\overrightarrow{AB}\right| = \sqrt{(-1)^2+(-2)^2+(-6)^2} = \sqrt{1+4+36} = \sqrt{41}$

$\Rightarrow \left|\overrightarrow{BC}\right| = \sqrt{(2)^2+(-1)^2+(1)^2} = \sqrt{1+4+1} = \sqrt{6}$

$\Rightarrow \left|\overrightarrow{AC}\right| = \sqrt{(-1)^2+(3)^2+(5)^2} = \sqrt{1+9+25} = \sqrt{35}$

Therefore,

$\left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{AC}\right|^2 = 6+35 = 41 = \left|\overrightarrow{AB}\right|^2$

Hence, Δ ABC is a right – angled triangle.

Q 18:

If $\vec{a}$ is a nonzero vector of magnitude 'a' and λ a nonzero scalar, then λ $\vec{a}$ is unit vector if

(A) λ = 1

(B) λ = – 1

(C) $a = |\lambda|$

(D) $a = \frac{1}{|\lambda|}$

Solution 18:

Vector $\lambda\vec{a}$ is a unit vector if $|\lambda\vec{a}| = 1$

Now,

$|\lambda\vec{a}| = 1$

$\Rightarrow |\lambda|\,|\vec{a}| = 1$

$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|}$ . . . . [λ ≠ 0]

$\Rightarrow a = \frac{1}{|\lambda|}$ . . . . [$|\vec{a}| = a$]

Therefore, vector $\lambda\vec{a}$ is a unit vector if $a = \frac{1}{|\lambda|}$

The correct answer is D.

Exercise 10.4

Q 1: Find $\left|\vec{a}\times\vec{b}\right|$, if $\vec{a} = \hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b} = 3\hat{i}-2\hat{j}+2\hat{k}$

Solution 1:

We have,

$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$\vec{a} \times \vec{b} = \hat{i}\,(-14 + 14) - \hat{j}\,(2 - 21) + \hat{k}\,(-2 + 21) = 19\hat{j} + 19\hat{k}$

Therefore,

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{(19)^2 + (19)^2} = \sqrt{2 \times (19)^2} = 19\sqrt{2}$$

Q 2: Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b} and \vec{a} - \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k} and \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

Solution 2:

We have,

$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k} and \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

Therefore,

$$\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}, \vec{a} - \vec{b} = 2\hat{i} + 4\hat{k} \ \left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} \left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right) = \hat{i}\,(16) - \hat{j}\,(16) + \hat{k}\,(-8) = 16\hat{i} - 16\hat{j} - 8\hat{k}$$

$$\left|\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right)\right| = \sqrt{16^2 + (-16)^2 + (-8)^2} \ \left|\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right)\right| = \sqrt{2^2 \times 8^2 + 2^2 \times 8^2 + 8^2} \ \left|\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right)\right| = 8\sqrt{2^2 + 2^2 + 1} = 8\sqrt{9} = 8 \times 3 = 24$$

Therefore, the unit vector perpendicular to each of the vectors $\vec{a} + \vec{b} and \vec{a} - \vec{b}$ is given by the relation,

$= \pm \frac{\left(\vec{a}+\vec{b}\right) \times \left(\vec{a}-\vec{b}\right)}{\left|\left(\vec{a}+\vec{b}\right) \times \left(\vec{a}-\vec{b}\right)\right|} = \pm \frac{16\hat{i} - 16\hat{j} - 8\hat{k}}{24}$

$= \pm \frac{2\hat{i} - 2\hat{j} - \hat{k}}{3} = \pm \frac{2}{3}\hat{i} \mp \frac{2}{3}\hat{j} \mp \frac{1}{3}\hat{k}$

Q 3: If a unit vector $\vec{a}$ makes angle $\frac{\pi}{3} with \hat{i}, \frac{\pi}{4} angle with \hat{j}$ and an acute angle θ with $\vec{k}$, then find θ and hence, the compounds of $\vec{a}$

Solution 3:

Let unit vector $\vec{a}$ have (a_1, a_2, a_3) componenets

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Since, $\vec{a}$ is a unit vector , $|\vec{a}| = 1$

Also, it is given that $\vec{a}$ makes angles $\frac{\pi}{3} with \hat{i}, \frac{\pi}{4} angle with \hat{j}$ and an acute angle θ with $\vec{k}$

Then, we have :

$\cos\frac{\pi}{3} = \frac{a_1}{|\vec{a}|}$

$\Rightarrow \frac{1}{2} = a_1 \ldots\ldots\ldots [|\vec{a}| = 1]$

$\cos\frac{\pi}{4} = \frac{a_2}{|\vec{a}|}$

$\Rightarrow \frac{1}{\sqrt{2}} = a_2 \ldots\ldots\ldots [|\vec{a}| = 1]$

Also , $\cos\theta = \frac{a_3}{|\vec{a}|}$

$\Rightarrow a_3 = \cos\theta$

$|a| = 1$

$\Rightarrow \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2} = 1$

$\Rightarrow \frac{1}{2}^2 + \frac{1}{\sqrt{2}}^2 + \cos^2\theta = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow \frac{3}{4} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{3}$

Therefore,

$a_3 = \cos\frac{\pi}{3} = \frac{1}{2}$

Therefore, $\theta = \frac{\pi}{3} and the components of \vec{a} are \left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

Q 4: Show that:

$\left(\vec{a}-\vec{b}\right) \times \left(\vec{a}+\vec{b}\right) = 2\left(\vec{a}\times\vec{b}\right)$

Solution:

To prove:

$\left(\vec{a}-\vec{b}\right) \times \left(\vec{a}+\vec{b}\right) = 2\left(\vec{a}\times\vec{b}\right)$

= $\left(\vec{a}-\vec{b}\right) \times \left(\vec{a}+\vec{b}\right) = \left(\vec{a}-\vec{b}\right)\times\vec{a} + \left(\vec{a}-\vec{b}\right)\times\vec{b}$ .... [By distributivity of vector product over addition]

= $\vec{a}\times\vec{a} - \vec{b}\times\vec{a} + \vec{a}\times\vec{b} - \vec{b}\times\vec{b}$ .... [again, by distributivity of vector product over addition]

= $\vec{0} + \vec{a}\times\vec{b} + \vec{a}\times\vec{b} - \vec{0}$

= $2\vec{a}\times\vec{b}$

Q 5: Find $\lambda and \mu if \left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right) = \vec{0}$

Solution:

$\left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right) = \vec{0}$

$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda\mu & \end{vmatrix}$ = $0\hat{i}+0\hat{j}+0\hat{k}$

$\Rightarrow \hat{i}\left(6\mu-27\mu\right) - \hat{j}\left(2\mu-27\right) + \hat{k}\left(2\lambda-6\right) = 0\hat{i}+0\hat{j}+0\hat{k}$

On comparing the corresponding components, we have :

$6\mu-27\lambda = 0$
$2\mu-27 = 0$
$2\lambda-6 = 0$

Now,

$2\lambda-6 = 0 \Rightarrow \lambda = 3$

$\Rightarrow 2\mu-27 = 0$
$\Rightarrow \mu = \frac{27}{2}$

Therefore, $\lambda = 3 and \mu = \frac{27}{2}$

Q 6: Given that:

$\vec{a}.\vec{b} = 0 and \vec{a}\times\vec{b} = \vec{0}$

What can you conclude about the vectors $\vec{a} and \vec{b}$ **?**

Solution:

$\vec{a}.\vec{b} = 0$

Then ,

1. **i)** either $\vec{a} = 0 or \vec{b} = 0, or \vec{a}\perp\vec{b}\left(in case \vec{a} and \vec{b} are non-zero\right)$

   $\vec{a}\times\vec{b} = 0$
2. **ii)** Either $\vec{a} = 0 or \vec{b} = 0, or \vec{a}\parallel\vec{b}\left(in case \vec{a} and \vec{b} are non-zero\right)$

   $\vec{a}\times\vec{b} = 0$

But, $\vec{a} and \vec{b}$ cannot be perpendicular and parallel simultaneously.

Therefore, $|\vec{a}| = 0 or \left|\vec{b}\right| = 0$

Q 7: Let the vectors $\vec{a},\vec{b},\vec{c} given as a_1\hat{i}+a_2\hat{j}+a_3\hat{k}, b_1\hat{i}+b_2\hat{j}+b_3\hat{k}, c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

Then show that = $\vec{a}\times\left(\vec{b}+\vec{c}\right) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

Solution:

We have,

$$a_1\hat{i}+a_2\hat{j}+a_3\hat{k}, b_1\hat{i}+b_2\hat{j}+b_3\hat{k}, c_1\hat{i}+c_2\hat{j}+c_3\hat{k}\left(\vec{b}+\vec{c}\right)=(b_1+c_1)\hat{i}+(b_2+c_2)\hat{j}+(b_3+c_3)\hat{k}$$

Now, $\vec{a}\times\left(\vec{b}+\vec{c}\right)\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1+c_1 & b_2+c_2 & b_3+c_3\end{vmatrix}$

$$=\hat{i}\left[a_2(b_3+c_3)-a_3(b_2+c_2)\right]-\hat{j}\left[a_1(b_3+c_3)-a_3(b_1+c_1)\right]+\hat{k}\left[a_1(b_2+c_2)-a_2(b_1+c_1)\right]$$

$$=\hat{i}\left[a_2b_3+a_2c_3-a_3b_2-a_3c_2\right]+\hat{j}\left[-a_1b_3-a_1c_3+a_3b_1+a_3c_1\right]+\hat{k}\left[a_1b_2+a_1c_2-a_2b_1-a_2c_1\right] \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\Rightarrow\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

$= \hat{i}\left[a_2b_3-a_3b_2\right]+\hat{j}\left[a_3b_1-a_1b_3\right]+\hat{k}\left[a_1b_2-a_2b_1\right]$ ……………………… **(2)**

$$\Rightarrow\vec{a}\times\vec{c}=\begin{vmatrix}\hat{I} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ c_1 & c_2 & c_3\end{vmatrix}$$

$\hat{i}\left[a_2c_3-a_3c_2\right]+\hat{j}\left[a_3c_1-a_1c_3\right]+\hat{k}\left[a_1c_2-a_2c_1\right]$ ……………………… **(3)**

On adding (2) and (3), we get:

$\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{c}\right)=\hat{i}\left[a_2b_3+a_2c_3+-a_3b_2-a_3c_2\right]+\hat{j}\left[a_3b_1+a_3c_1+-a_1b_3-a_1c_3\right]+\hat{k}\left[a_1b_2+a_1c_2+-a_2b_1-a_2c_1\right]$ ……………………… **(4)**

Now, from (1) and (4), we have:

$$\vec{a}\times\left(\vec{b}+\vec{c}\right)=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$$

Hence, the given result is proved.

Question 8: Suppose, any of the vector $\overrightarrow{m}\ or\ \overrightarrow{n}=\overrightarrow{0}$, then $\overrightarrow{m}\times\overrightarrow{n}=\overrightarrow{0}$.

Is the above given statement true?

Give reason in support to your answer with an example.

Answer 8:

By taking any two non – zero vectors, for the condition $\overrightarrow{m}\times\overrightarrow{n}=\overrightarrow{0}$.

Suppose, $\overrightarrow{m}=\hat{i}+2\hat{j}+3\hat{k}\ and\ \overrightarrow{n}=2\hat{i}+4\hat{j}+6\hat{k}$

$Then,$

$$\overrightarrow{PQ}\times\overrightarrow{QR}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 2 & 4 & 6\end{vmatrix}\hat{i}(4-4)-\hat{j}(6-6)+\hat{k}(4-4)=0\hat{i}+0\hat{j}+0\hat{k}=\hat{0}$$

$Now,\ we\ find\ that,$

$$\left[\overrightarrow{m}\right]=\sqrt{(1)^2+(2)^2+(3)^2}=\sqrt{14}$$

$$So,\ \overrightarrow{m}\neq\overrightarrow{0}$$

$$\left[\overrightarrow{n}\right]=\sqrt{(2)^2+(4)^2+(6)^2}=\sqrt{56}$$

$$So,\ \overrightarrow{n}\neq\overrightarrow{0}$$

Thus, the above given statement is not true.

Question 9: P (1, 1, 2), Q (2, 3, 5) and R (1, 5, 5) are the vertices of a triangle. Obtain the area.

Answer 9:

Given:

Vertices of a triangle P (1, 1, 2), Q (2, 3, 5) and R (1, 5, 5)

The contiguous sides $\overrightarrow{PQ}\ and\ \overrightarrow{QR}$ of the triangle is given as,

$$\overrightarrow{PQ}=(2-1)\hat{i}+(3-1)\hat{j}+(5-2)\hat{k}=\hat{i}+2\hat{j}+3\hat{k}\ and\ \overrightarrow{QR}=(1-2)\hat{i}+(5-3)\hat{j}+(5-5)\hat{k}=-\hat{i}+2\hat{j}$$

$$Area\ of\ \Delta\ triangle=\frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{QR}\right|$$

$$\overrightarrow{PQ}\times\overrightarrow{QR}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ -1 & 2 & 0\end{vmatrix}$$

$|-1 \quad 2 \quad 0|$

$\hat{i}(-6) - \hat{j}(3) + \hat{k}(2+2) = -6\hat{i} - 3\hat{j} + 4\hat{k}$

$\left|\overrightarrow{PQ} \times \overrightarrow{QR}\right| = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{36+9+16} = \sqrt{61}$

Thus, $\frac{\sqrt{61}}{2}$ is the area of triangle ABC.

Question 10: $\overrightarrow{m} = \hat{i} - \hat{j} + 3\hat{k}\ and\ \overrightarrow{n} = 2\hat{i} - 7\hat{j} + \hat{k}$ are the adjacent sides of a vector. Obtain the area of the parallelogram.

Answer 10:

The area of the parallelogram whose contiguous sides are $\overrightarrow{m}\ and\ \overrightarrow{n}\, is \left|\overrightarrow{m} \times \overrightarrow{n}\right|$

Given,

$\overrightarrow{m} = \hat{i} - \hat{j} + 3\hat{k}\ and\ \overrightarrow{n} = 2\hat{i} - 7\hat{j} + \hat{k}$ are the adjacent sides of a vector.

$$\overrightarrow{m} \times \overrightarrow{n} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$\hat{i}(-1+21) - \hat{j}(1-6) + (-7+2)\hat{k} = 20\hat{i} + 5\hat{j} - 5\hat{k}$

$\left|\overrightarrow{m} \times \overrightarrow{n}\right| = \sqrt{20^2 + 5^2 + 5^2} = \sqrt{400+25+25} = 15\sqrt{2}$

$15\sqrt{2}$ is the area of the given parallelogram.

Question 11: Suppose, vectors $\overrightarrow{m}\ and\ \overrightarrow{n}\ in\ such\ a\ way\ that \left|\overrightarrow{m}\right| = 3\ and \left|\overrightarrow{n}\right| = \frac{\sqrt{2}}{3},\ then\ \overrightarrow{m} \times \overrightarrow{n}$ is a unit vector, suppose the angle between the two vectors is

$(i)\ \frac{\pi}{6},\ (ii)\ \frac{\pi}{4},\ (iii)\ \frac{\pi}{3}\ (iv) \frac{\pi}{2}$

Answer 11:

Given, $\left|\overrightarrow{m}\right| = 3\ and \left|\overrightarrow{n}\right| = \frac{\sqrt{2}}{3},\ then\ as\ we\ know\ \overrightarrow{m} \times \overrightarrow{n} = 1$, where s is a unit vector perpendicular to both $\overrightarrow{m}\ and\ \overrightarrow{n}$ and Θ is the angle between $\overrightarrow{m}\ and\ \overrightarrow{n}$

$$\overrightarrow{m} \times \overrightarrow{n} = \left|\overrightarrow{m}\right| \left|\overrightarrow{n}\right|\ sin\theta\hat{s}$$

Now, $\overrightarrow{m}\ and\ \overrightarrow{n}\ is\ a\ unit\ vector\ if\ \left|\overrightarrow{m} \times \overrightarrow{n}\right| = 1$

$$\left|\overrightarrow{m} \times \overrightarrow{n}\right| = 1$$

$$\left|\left|\overrightarrow{m}\right| \left|\overrightarrow{n}\right| sin\ \Theta\hat{s}\right| = 1$$

$$\left|\left|\overrightarrow{m}\right| \left|\overrightarrow{n}\right| sin\ \Theta\right| = 1$$

$$3 \times \frac{\sqrt{2}}{3} \times sin\ \Theta = 1$$

$$sin\ \Theta = \frac{1}{\sqrt{2}}$$

$$\Theta = \frac{\pi}{4}$$

The correct answer is option (ii)

Question 12: The area of the rectangle with P, Q, R and S as the vertices with positive vectors
$-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}\ and - \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}\ respectively\ is$

$(i)\frac{1}{2}, \quad (ii)1$

$(iii)2, \quad (iv)4$

Answer 12:

Given,

The area of the rectangle PQRS with P, Q, R and S as the vertices with positive vectors such as:

$\overrightarrow{OP} = -\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k},\ \overrightarrow{OQ} = \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k},\ \overrightarrow{OR} = \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}\ and\ \overrightarrow{OS} - \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$

The contiguous sides $\overrightarrow{PQ}\ and\ \overrightarrow{QR}$ of the rectangle is given as,

$$\overrightarrow{PQ} = (1+1)\hat{i} + \left(\tfrac{1}{2} - \tfrac{1}{2}\right)\hat{j} + (4-4)\hat{k} = 2\hat{i}\ and\ \overrightarrow{QR} = (1-1)\hat{i} + \left(-\tfrac{1}{2} - \tfrac{1}{2}\right)\hat{j} + (4-4)\hat{k} = -\hat{j}\ \overrightarrow{PQ} \times \overrightarrow{QR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\hat{k}(-2) = -2\hat{k}$$

$$\left|\overrightarrow{PQ} \times \overrightarrow{QR}\right| = \sqrt{(-2)^2} = 2$$

Area of a rectangle is 2 square units.

Option (iii) is the correct answer.