NCERT SOLUTIONS
CLASS-XII MATHS
CHAPTER-7
INTEGRALS

Exercise – 7.1

Question 1:

By the method of inspection obtain an integral (or anti – derivative) of the sin 3x.

Answer:

As the derivative is sin 3x and x is the function of the anti – derivative of sin 3x.

$\frac{d}{dx}(cos\ 3x) = -3sin\ 3x$
$sin\ 3x = -\frac{1}{3}\frac{d}{dx}(cos\ 3x)$
$sin\ 3x = \frac{d}{dx}(-\frac{1}{3}cos\ 3x)$
$Hence,\ the\ anti-derivative\ of\ sin\ 3x\ is\ (-\frac{1}{3}cos\ 3x)$

Question 2:

By the method of inspection obtain an integral (or anti – derivative) of the cos 2x.

Answer:

As the derivative is cos 2x and x is the function of the anti – derivative of cos 2x

$\frac{d}{dx}(sin\ 2x) = -2cos\ 2x$
$cos\ 2x = \frac{1}{2}\frac{d}{dx}(sin\ 2x)$
$cos\ 2x = \frac{d}{dx}(\frac{1}{2}(sin\ 2x))$
$Hence,\ the\ anti-derivative\ of\ sin\ 2x\ is\ (-\frac{1}{2}cos\ 2x)$

Question 3:

By the method of inspection obtain an integral (or anti – derivative) of the e^{5x}.

Answer:

As the derivative is e^{5x} and x is the function of the anti – derivative of e^{5x}

$\frac{d}{dx}(e^{5x}) = 5e^{5x}$
$e^{5x} = \frac{1}{5}\frac{d}{dx}(e^{5x})$
$e^{5x} = \frac{d}{dx}(\frac{1}{5}e^{5x})$
$Hence,\ the\ anti-derivative\ of\ e^{5x} is \frac{1}{5}e^{5x}$

Question 4:

By the method of inspection obtain an integral (or anti – derivative) of the $(mx + n)^2$.

Answer:

As the derivative is $(mx + n)^2$ and x is the function of the anti – derivative of $(mx + n)^2$

$\frac{d}{dx}(mx+n)^3 = 3m(mx+n)^2$
$(mx+n)^2 = \frac{1}{3m}\frac{d}{dx}(mx+n)^3$
$(mx+n)^2 = \frac{d}{dx}(\frac{1}{3m}(mx+n)^3)$
$Hence,\ the\ anti-derivative\ of\ (mx+n)^2 is\ \frac{1}{3m}(mx+n)^3$

Question 5:

By the method of inspection obtain an integral (or anti – derivative) of the sin 3x – 5 e^{2x}

Answer:

As the derivative is (sin 3x – 5 e^{2x}) and x is the function of the anti – derivative of (sin 3x – 5 e^{2x})

$\frac{d}{dx}(-\frac{1}{3}cos\ 3x - \frac{5}{2}e^{2x}) = sin3x - 5e^{2x}$
$Hence,\ the\ anti-derivative\ of\ sin\ 3x - 5e^{2x}\ is\ (-\frac{1}{3}cos\ 3x - \frac{5}{2}e^{2x})$

Question 6:

By the method of inspection obtain an integral of the $\int(4e^{2u}+1)du$

Answer:

Integral of $(4e^{2u}+1)$ and u is the function of the integral $(4e^{2u}+1)$.

$\int(4e^{2u}+1)du$
$4\int e^{2u}du+\int 1du$
$4(\frac{e^{2u}}{2})+u+c$
$2e^{2u}+u+c$
$Where\ c\ is\ the\ constant.$

Question 7:

By the method of inspection obtain an integral of the $\int u^2(1-\frac{1}{u^2})du$

Answer:

Integral of $u^2(1-\frac{1}{u^2})$ and u is the function of the integral $u^2(1-\frac{1}{u^2})$

$\int u^2(1-\frac{1}{u^2})du$
$\int(u^2-1)du$
$\frac{u^3}{3}-u+c$
$Where\ c\ is\ the\ constant$

Question 8:

By the method of inspection obtain an integral of the $\int(au^2+bu+c)du$

Answer:

Integral of au^2+bu+c and u is the function of the integral au^2+bu+c

$\int(au^2+bu+c)du$
$a\int(u^2)du+b\int udu+c\int 1du$
$a(\frac{u^3}{3})+b(\frac{u^2}{2})+cu+C$
$Where\ C\ is\ the\ constant$

Question 9:

By the method of inspection obtain an integral of the $\int(au^2+e^u)du$

Answer:

Integral of au^2+e^u and u is the function of the integral au^2+e^u

$\int(au^2+e^u)du$
$a\int(u^2)du+\int e^u du$
$a(\frac{u^3}{3})+e^u+C$
$Where\ C\ is\ the\ constant$

Question 10:

By the method of inspection obtain an integral of the $\int(\sqrt{u}+\frac{1}{\sqrt{u}})^2du$

Answer:

Integral of $(\sqrt{u}+\frac{1}{\sqrt{u}})^2$ and u is the function of the integral $(\sqrt{u}+\frac{1}{\sqrt{u}})^2$

$(\sqrt{u}+\frac{1}{\sqrt{u}})^2$
$\int(u+\frac{1}{u}-2)du$
$\int udu+\int\frac{1}{u}du-2\int 1du$
$\frac{u^2}{2}+log|u|-2u+C$
$Where\ C\ is\ the\ constant$

Question 11:

By the method of inspection obtain an integral of the $\int\frac{u^3+4u^2+4}{u^2}du$

Answer:

Integral of and u is the function of the integral $\frac{u^3+4u^2+4}{u^2}$

$\int\frac{u^3+4u^2+4}{u^2}du$
$\int udu+4\int 1du+\int\frac{4}{u^2}du$
$\frac{u^2}{2}+4u+\frac{4}{x}+C$
$Where\ C\ is\ the\ constant$

Question 12:

By the method of inspection obtain an integral of the $\frac{u^3+4u+4}{}$

By the method of inspection obtain an integral of the $\frac{}{\sqrt{u}}$

Answer:

Integral of $\frac{u^3+4u+4}{\sqrt{u}}$ and u is the function of the integral $\frac{u^3+4u+4}{\sqrt{u}}$

$\int \frac{u^3+4u+4}{\sqrt{u}} du$
$\int (u^{\frac{5}{2}}+4u^{\frac{1}{2}}+4u^{-\frac{1}{2}})$
$= \frac{u^{\frac{7}{2}}}{\frac{7}{2}}+\frac{4(u^{\frac{3}{2}})}{\frac{3}{2}}+\frac{4(u^{\frac{1}{2}})}{\frac{1}{2}}+C$
$= \frac{2}{7}(u^{\frac{7}{2}})+\frac{8}{3}(u^{\frac{3}{2}})+8u^{\frac{1}{2}}+C$
$= \frac{2}{7}(u^{\frac{7}{2}})+\frac{8}{3}(u^{\frac{3}{2}})+8\sqrt{u}+C$
$Where\ C\ is\ the\ constant$

Question 13:

By the method of inspection obtain an integral of the $\frac{u^3-u^2+u+1}{u-1}$

Answer:

Integral of $\frac{u^3-u^2+u+1}{u-1}$ and u is the function of the integral $\frac{u^3-u^2+u+1}{u-1}$

$\int \frac{u^3-u^2+u+1}{u-1} du$
$On\ dividing,\ we\ get$
$\int (u^2+1) du$
$\int u^2 du+\int 1 du$
$\frac{u^3}{3}+u+CWhere\ C\ is\ the\ constant$

Question 14:

By the method of inspection obtain an integral of the $(1-u)\sqrt{u}$

Answer:

Integral of $(1-u)\sqrt{u}$ and u is the function of the integral $(1-u)\sqrt{u}$

$\int (1+u)\sqrt{u}\ du$
$\int (\sqrt{u}+u^{\frac{3}{2}}) du$
$\int u^{\frac{1}{2}} du+\int u^{\frac{3}{2}} du$
$\frac{u^{\frac{3}{2}}}{\frac{3}{2}}+\frac{u^{\frac{5}{2}}}{\frac{5}{2}}+C$
$\frac{2}{3}u^{\frac{3}{2}}+\frac{2}{5}u^{\frac{5}{2}}+C$
$Where\ C\ is\ the\ constant$

Question 15:

By the method of inspection obtain an integral of the $\sqrt{u}(3u^2+2u+5)$

Answer:

Integral of $\sqrt{u}(3u^2+2u+5)$ and u is the function of the integral $\sqrt{u}(3u^2+2u+5)$

$\int \sqrt{u}(3u^2+2u+5) du$
$\int (3u^{\frac{5}{2}}+2u^{\frac{3}{2}}+5u^{\frac{1}{2}}) du$
$3\int u^{\frac{5}{2}} du+2\int u^{\frac{3}{2}} du+5\int u^{\frac{1}{2}} du$
$3(\frac{u^{\frac{7}{2}}}{\frac{7}{2}})+2(\frac{u^{\frac{5}{2}}}{\frac{5}{2}})+5(\frac{u^{\frac{3}{2}}}{\frac{3}{2}})+C$
$\frac{6}{7}u^{\frac{7}{2}}+\frac{4}{5}u^{\frac{5}{2}}+\frac{10}{3}u^{\frac{3}{2}}+C$
$Where\ C\ is\ the\ constant$

Question 16:

By the method of inspection obtain an integral of the $2u-2cos\ u+e^u$

Answer:

Integral of $2u-2cos\ u+e^u$ and u is the function of the integral $2u-2cos\ u+e^u$

$\int (2u-2cos\ u+e^u) du$
$2\int u du-2\int cos\ u du+\int e^u du$
$2\frac{u^2}{2}-2(sinu)+e^u+C$
$u^2-2sin\ u+e^u+CWhere\ C\ is\ the\ constant$

Question 17:

By the method of inspection obtain an integral of the $(4v^2+2sinv+6\sqrt{v})$

Answer:

Integral of $(4v^2+2sinv+6\sqrt{v})$ and v is the function of the integral $(4v^2+2sinv+6\sqrt{v})$

$\int(4v^2+2sinv+6\sqrt{v})\ dv$
$4\int v^2dv+2\int sinvdv+6\int v^{\frac{1}{2}}$
$\frac{4v^3}{3}+2(-cos\ v)+6(\frac{v^{\frac{3}{2}}}{\frac{3}{2}})+C$
$\frac{4}{3}v^3-2cos\ v+4v^{\frac{3}{2}}+C$
$Where\ C\ is\ the\ constant$

Question 18:

By the method of inspection obtain an integral of the $sec\ \Theta(tan\ \Theta+sec\ \Theta)$

Answer:

Integral of $sec\ \Theta(tan\ \Theta+sec\ \Theta)$ and Θ is the function of the integral $sec\ \Theta(tan\ \Theta+sec\ \Theta)$

$\int sec\ \Theta(tan\ \Theta+sec\ \Theta)d\Theta$
$\int(sec\ \Theta\ tan\ \Theta+sec^2\ \Theta)d\Theta$
$sec\ \Theta\ +tan\ \Theta+K$
$Where\ K\ is\ the\ constant$

Question 19:

By the method of inspection obtain an integral of the $\frac{sec^2\ \Theta}{cosec^2\ \Theta}$

Answer:

Integral of $\frac{sec^2\ \Theta}{cosec^2\ \Theta}$ and $\frac{3-2sin\ \Theta}{cos^2\ \Theta}$c is the function of the integral $\frac{sec^2\ \Theta}{cosec^2\ \Theta}$

$\int\frac{sec^2\ \Theta}{cosec^2\ \Theta}d\Theta$
$\int\frac{\frac{1}{cos^2\ \Theta}}{\frac{1}{sin^2\ \Theta}}d\Theta$
$\int\frac{sin^2\ \Theta}{cos^2\ \Theta}d\Theta$
$\int(tan^2\ \Theta)d\Theta$
$\int(sec^2\ \Theta-1)d\Theta$
$\int sec^2\ \Theta d\Theta-\int 1d\Theta$
$tan\Theta-\Theta+K$
$Where\ K\ is\ the\ constant$

Question 20:

By the method of inspection obtain an integral of the $\frac{3-2sin\ \Theta}{cos^2\ \Theta}$

Answer:

Integral of $\frac{3-2sin\ \Theta}{cos^2\ \Theta}$ and $\frac{3-2sin\ \Theta}{cos^2\ \Theta}$ is the function of the integral $\frac{3-2sin\ \Theta}{cos^2\ \Theta}$

$\int\frac{3-2sin\ \Theta}{cos^2\ \Theta}d\Theta$
$\int(\frac{3}{cos^2\ \Theta}-\frac{2sin\ \Theta}{cos^2\ \Theta})d\Theta$
$3\int sec^2\ \Theta\ d\Theta-2\int tan\ \Theta\ sec\ \Theta d\Theta$
$3tan\ \Theta-2sec\ \Theta+K$
$Where\ K\ is\ the\ constant$

Question 21:

Which of the following below is an integral of $\sqrt{u}+\frac{1}{\sqrt{u}}$:

$(a)\frac{1}{3}u^{\frac{1}{3}}+2u^{\frac{1}{2}}+C$
$(b)\frac{2}{3}u^{\frac{2}{3}}+\frac{1}{2}u^2+C$
$(c)\frac{2}{3}u^{\frac{3}{2}}+2u^{\frac{1}{2}}+C$
$(d)\frac{3}{2}u^{\frac{3}{2}}+\frac{1}{2}u^{\frac{1}{2}}+C$

Answer:

Integral of $\sqrt{u}+\frac{1}{\sqrt{u}}$ and u is the function of the integral $\sqrt{u}+\frac{1}{\sqrt{u}}$

$\int\sqrt{u}+\frac{1}{\sqrt{u}}du$
$\int u^{\frac{1}{2}}du+\int u^{-\frac{1}{2}}du$
$\frac{u^{\frac{3}{2}}}{\ }+\frac{u^{\frac{1}{2}}}{\ }+C$

$\frac{3}{2}u^{\frac{3}{2}} + 2u^{\frac{1}{2}}$

$Option\ c\ is\ correct$

Question 22:

$Suppose\ \frac{d}{dr}f(r) = 4r^3 - \frac{3}{r^4},\ in\ such\ a\ way\ that\ f(2) = 0,\ then\ f(r)\ is$

$(a)r^4 + \frac{1}{r^3} - \frac{129}{8}\ (b)r^3 + \frac{1}{r^4} + \frac{129}{8}\ (c)r^4 + \frac{1}{r^3} + \frac{129}{8}\ (d)r^3 + \frac{1}{r^4} - \frac{129}{8}$

Answer:

Given,

$\frac{d}{dr}f(r) = 4r^3 - \frac{3}{r^4}$

$Integral\ of\ 4r^3 - \frac{3}{r^4} = f(r)$

$f(r) = \int 4r^3 - \frac{3}{r^4}\ dr$

$f(r) = 4\int r^3 dr - 3\int (r^{-4}) dr$

$f(r) = 4\frac{r^4}{4} - 3\frac{r^{-3}}{-3} + K$

$f(r) = r^4 + \frac{1}{r^3} + K$

$And,$

$f(2) = 0$

$f(2) = 2^4 + \frac{1}{2^3} + K = 0$

$16 + \frac{1}{8} + K = 0$

$K = -\frac{129}{8}$

$f(r) = r^4 + \frac{1}{r^3} - \frac{129}{8}$

$Option\ (a)\ is\ correct$

Exercise 7.2

Question 1:

Obtain an integral (or anti – derivative) of the $\frac{2u}{1+u^2}$

Answer:

Suppose, $1 + u^2 = z$

2u du = dz

$\int \frac{2u}{1+u^2} = \int \frac{1}{z}\ dz$

$log\,|z| + K$

$log\,|1 + u^2| + K$

$log(1 + u^2) + K$

Question 2:

Obtain an integral (or anti – derivative) of the $\frac{(log\ u)^2}{u}$

Answer:

Suppose, $log\,|u| = z$

$log\,|u| = z$

$\frac{1}{u}du = dz$

$\int \frac{(log|u|)^2}{u}du = \int z^2 dz$

$= \frac{z^3}{3} + C$

$= \frac{(log|u|)^3}{3} + C$

Question 3:

Obtain an integral (or anti – derivative) of the $\frac{1}{u+u\ log\ u}$

Answer:

$\frac{1}{u+u\ log\ u} = \frac{1}{u(1+log\ u)}$

Suppose, 1 + log u = z

$\frac{1}{u}du = dz$

$$\int \frac{1}{u(1+log\ u)}du = \int \frac{1}{z}dz = log\,|z| + C$$
$$= log\,|1 + logu| + C$$

Question 4:

Obtain an integral (or anti – derivative) of the $sin\ u.\,sin(cos\ u)$

Answer:

$sin\ u.\,sin(cos\ u)$

Suppose, cos u = x

– sin u du = dx

$$\int sin\ u.\,sin(cos\ u)du = -\int sin\ xdx$$
$$= -[-cosx] + C$$
$$= cosx + C$$
$$= cos(cosu) + C$$

Question 5:

Obtain an integral (or anti – derivative) of the $sin\ (mr + n)cos\ (mr + n)$

Answer:

Suppose, $sin\ (mr + n)cos\ (mr + n) = \frac{2sin\ (mr+n)cos\ (mr+n)}{2} = \frac{sin2(mr+n)}{2}$

$$Suppose\ 2(mr + n) = z$$
$$2mdr = dz$$
$$\int \frac{sin2(mr+n)}{2}dr = \frac{1}{2}\int \frac{sin\ z\ dz}{2m}$$
$$= \frac{1}{4m}[-cosz] + C$$
$$= -\frac{1}{4m}cos2(mr + n) + C$$

Question 6:

Obtain an integral (or anti – derivative) of the $\sqrt{mr + n}$

Answer:

Suppose, mr + n = z

m dr = dz

$$dr = \frac{1}{m}dz$$
$$\int(mr + n)^{\frac{1}{2}}dr = \frac{1}{m}\int z^{\frac{1}{2}}dz$$
$$\frac{1}{m}\left(\frac{z^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$\frac{2}{3m}(mr + n)^{\frac{3}{2}} + C$$

Question 7:

Obtain an integral (or anti – derivative) of the $u\sqrt{u + 2}$

Answer:

Suppose, u + 2 = z

du = dz

$$\int u\sqrt{u + 2}du = \int(z - 2)\sqrt{z}dz$$
$$= \int(z^{\frac{3}{2}} - 2z^{\frac{1}{2}})dz$$
$$= \int z^{\frac{3}{2}}dz - 2\int z^{\frac{1}{2}})dz$$
$$= \frac{z^{\frac{5}{2}}}{\frac{5}{2}} - 2\frac{z^{\frac{3}{2}}}{\frac{3}{2}} + C$$
$$= \frac{2}{5}z^{\frac{5}{2}} - \frac{4}{3}z^{\frac{3}{2}} + C$$
$$= \frac{2}{5}(x + 2)^{\frac{5}{2}} - \frac{4}{3}(x + 2)^{\frac{3}{2}} + C$$

Question 8:

Obtain an integral (or anti – derivative) of the $u\sqrt{1 + 2u^2}$

Answer:

Suppose, $1 + 2\ u^2 = z$

4u du = dz

$$\int u\sqrt{1 + 2u^2}du = \int \frac{\sqrt{z}}{4}dz$$

$= \frac{1}{4}\int z^{\frac{1}{2}} dz$

$= \frac{1}{4}(\frac{z^{\frac{3}{2}}}{\frac{3}{2}}) + C$

$= \frac{1}{6}(1 + 2u^2)^{\frac{3}{2}} + C$

Question 9:

Obtain an integral (or anti – derivative) of the $(4u + 2)\sqrt{u^2 + u + 1}$

Answer:

Suppose, $u^2 + u + 1 = z$

$(2u + 1)\, du = dz$

$\int(4u + 2)\sqrt{u^2 + u + 1}du = \int 2\sqrt{z}dz$
$= 2\int \sqrt{z}dz$

$= 2(\frac{z^{\frac{3}{2}}}{\frac{3}{2}}) + C$

$= \frac{4}{3}(u^2 + u + 1)^{\frac{3}{2}} + C$

Question 10:

Obtain an integral (or anti – derivative) of the $\frac{1}{u-\sqrt{u}}$

Answer:

$\frac{1}{u-\sqrt{u}} = \frac{1}{\sqrt{u}(\sqrt{u}-1)}$

Suppose,

$\sqrt{u}-1 = z\frac{1}{2\sqrt{u}}du = dz$
$\int \frac{1}{\sqrt{u}(\sqrt{u}-1)}du = \int \frac{2}{z}dz$
$2log\,|z| + C$
$2log\,|\sqrt{u}-1| + C$

Question 11:

Obtain an integral (or anti – derivative) of the $\frac{u}{\sqrt{u+4}}$, x > 0

Answer:

Suppose, $u + 4 = r$

$du = dr$

$\int \frac{u}{\sqrt{u+4}}du = \int \frac{(r-4)}{\sqrt{r}}dr$
$= \int(\sqrt{r}-\frac{4}{\sqrt{r}})dr$

$= \frac{r^{\frac{3}{2}}}{\frac{3}{2}}-4(\frac{r^{\frac{1}{2}}}{\frac{1}{2}}) + C$

$= \frac{2}{3}r^{\frac{3}{2}}-8r^{\frac{1}{2}} + C$

$= \frac{2}{3}r.\,r^{\frac{1}{2}}-8r^{\frac{1}{2}} + C$

$= \frac{2}{3}r^{\frac{1}{2}}(r-12) + C$

$= \frac{2}{3}(u + 4)^{\frac{1}{2}}(u + 4-12) + C$

$= \frac{2}{3}\sqrt{(u + 4)}(u-8) + C$

Question 12:

Obtain an integral (or anti – derivative) of the $(u^3-1)^{\frac{1}{3}}u^5$

Answer:

Suppose, $u^3 - 1 = r$

$3u^2 = dr$

$\int(u^3-1)^{\frac{1}{3}}u^5du = \int(u^3-1)^{\frac{1}{3}}u^3.\,u^2du$
$= \int r^{\frac{1}{3}}(r + 1)\frac{dr}{3}$
$= \frac{1}{3}\int(r^{\frac{4}{3}} + r^{\frac{1}{3}})dr$

$= \frac{1}{3}[\frac{r^{\frac{7}{3}}}{\frac{7}{3}} + \frac{r^{\frac{4}{3}}}{\frac{4}{3}}] + C$

$= \frac{1}{3}[\frac{3}{7}r^{\frac{7}{3}} + \frac{3}{4}r^{\frac{4}{3}}] + C$

$= \frac{1}{7}(u^3-1)^{\frac{7}{3}} + \frac{1}{4}(u^3-1)^{\frac{4}{3}}] + C$

Question 13:

Obtain an integral (or anti – derivative) of the $\frac{u^2}{(2+3u^3)^3}$

Answer:

Suppose, $2 + 3u^3 = z$

$9u^2du = dz$

$\int \frac{u^2}{(2+3u^3)} du = \frac{1}{9} \int \frac{dz}{(z)^3}$

$= \frac{1}{9} \int (z)^{-3} dz$

$= \frac{1}{9}(\frac{z^{-2}}{-2}) + C$

$=- \frac{1}{18}(\frac{1}{z^2}) + C$

$= \frac{-1}{18(2+3u^3)^2} + C$

Question 14:

Obtain an integral (or anti – derivative) of the $\frac{1}{u(logu)^n}, x > 0$

Answer:

Suppose, $logu = z$

$\frac{1}{u} du = dz$

$\int \frac{1}{u(logu)^n} du = \int \frac{dz}{z^n}$

$= \int z^{-n} dz$

$= \frac{z^{-n+1}}{-n+1} + C$

$= \frac{z^{1-n}}{1-n} + C$

$= \frac{x^{1-n}}{1-n} + C$

Question 15:

Obtain an integral (or anti – derivative) of the $\frac{u}{9-4u^2}$

Answer:

Suppose, $9-4u^2 = r$

$-8udu = dr$

$\int \frac{u}{9-4u^2} =- \frac{1}{8} \int \frac{1}{r} dr$

$=- \frac{1}{8} log|r| + C$

$=- \frac{1}{8} log|9-4u^2| + C$

Question 16:

Obtain an integral (or anti – derivative) of the e^{2m+3}

Answer:

Suppose, $2m + 3 = r$

$2dm = dr$

$\int e^{2m+3} dm = \frac{1}{2} \int e^r dr$

$= \frac{1}{2}(e^r) + C$

$= \frac{1}{2}(e^{2m+3}) + C$

Question 17:

Obtain an integral (or anti – derivative) of the $\frac{u}{e^{u^2}}$

Answer:

Suppose, u ^{2} = z

2u du = dz

$\int \frac{u}{e^{u^2}} du = \frac{1}{2} \int \frac{1}{e^z} dz$

$= \frac{1}{2} \int e^{-z} dz$

$= \frac{1}{2} \frac{e^{-z}}{-1} + C$

$=- \frac{1}{2} e^{-u^2} + C$

$=- \frac{1}{2e^{u^2}} + C$

Question 18:

Question 18:

Obtain an integral (or anti – derivative) of the $\frac{e^{tan^{-1}\Theta}}{1+\Theta^2}$

Answer:

Suppose, $tan^{-1}\Theta = z\frac{1}{1+\Theta^2}d\Theta = dz$

$$\int \frac{e^{tan^{-1}\Theta}}{1+\Theta^2}d\Theta = \int e^z dz$$
$$= e^z + C$$
$$= e^{tan^{-1}\Theta} + C$$

Question 19:

Obtain an integral (or anti – derivative) of the $\frac{e^{2u}-1}{e^{2u}+1}$

Answer:

$\frac{e^{2u}-1}{e^{2u}+1}$

Dividing the numerator and denominator by e ^{u}, we get

$$\frac{\frac{e^{2u}-1}{e^u}}{\frac{e^{2u}+1}{e^u}} = \frac{e^u-e^{-u}}{e^u+e^{-u}}$$

Suppose,

$$e^u + e^{-u} = z$$
$$(e^u - e^{-u})du = dz$$
$$\int \frac{e^{2u}-1}{e^{2u}+1}du = \int \frac{e^u-e^{-u}}{e^u+e^{-u}}du$$
$$= \int \frac{dz}{z}$$
$$= log\,|z| + C$$
$$= log\,|e^u + e^{-u}| + C$$

Question 20:

Obtain an integral (or anti – derivative) of the $\frac{e^{2u}-e^{-2u}}{e^{2u}+e^{-2u}}$

Answer:

Suppose, $e^{2u} + e^{-2u} = z$

$$(2e^{2u} - 2e^{-2u})du = dz$$
$$2(e^{2u} - e^{-2u})du = dz$$
$$\int \frac{e^{2u}-e^{-2u}}{e^{2u}+e^{-2u}} = \int \frac{dz}{2z}dz$$
$$= \frac{1}{2}\int \frac{1}{z}dz$$
$$= \frac{1}{2}log\,|z| + C$$
$$= \frac{1}{2}log\,|e^{2u} + e^{-2u}| + C$$

Question 21:

Obtain an integral (or anti – derivative) of the $tan^2(2\Theta-3)$

Answer:

$$tan^2(2\Theta-3) = sec^2(2\Theta-3)-1$$

Suppose, $2\Theta - 3 = z$

$$2d\Theta = dz$$
$$\int tan^2(2\Theta-3)d\Theta = \int[sec^2(2\Theta-3)-1]d\Theta$$
$$= \frac{1}{2}\int (sec^2 z)dz - \int 1d\Theta$$
$$= \frac{1}{2}tanz - \Theta + C$$
$$= \frac{1}{2}tan(2\Theta-3) - \Theta + C$$

Question 22:

Obtain an integral (or anti – derivative) of the $sec^2(7-4\theta)$

Answer:

Suppose, $(7-4\theta) = z$

$$-4\,d\theta = dz$$
$$\int sec^2(7-4\Theta)d\theta = -\frac{1}{4}\int sec^2 z dz$$
$$= -\frac{1}{4}(tanz) + C$$
$$= -\frac{1}{4}[tan(7-4\theta)] + C$$

Question 23:

Obtain an integral (or anti – derivative) of the $\frac{sin^{-1}\theta}{\sqrt{1-\theta^2}}$

Answer:

Suppose, $sin^{-1}\theta = z$

$$\frac{1}{\sqrt{1-\theta^2}}d\theta = dz$$
$$\int \frac{sin^{-1}\theta}{\sqrt{1-\theta^2}}d\theta = \int zdz$$
$$= \frac{z^2}{2} + C$$
$$= \frac{(sin^{-1}\theta)^2}{2} + C$$

Question 24:

Obtain an integral (or anti – derivative) of the $\frac{2cos\ \theta - 3sin\ \theta}{6cos\ \theta + 4sin\ \theta}$

Answer:

$$\frac{2cos\ \theta - 3sin\ \theta}{6cos\ \theta + 4sin\ \theta} = \frac{2cos\ \theta - 3sin\ \theta}{2(3cos\ \theta + 2sin\ \theta)}$$

Suppose,

$$3\ cos\ \theta + 2\ sin\ \theta = z$$
$$(-3\ sin\ \theta + 2\ cos\ \theta)d\theta = dz$$
$$\int \frac{2cos\ \theta - 3sin\ \theta}{6cos\ \theta + 4sin\ \theta}d\theta = \int \frac{dz}{2z}$$
$$= \frac{1}{2}\frac{1}{z}dz$$
$$= \frac{1}{2}log\,|z| + C$$
$$= \frac{1}{2}log\,|3\ cos\ \theta + 2\ sin\ \theta| + C$$

Question 25:

Obtain an integral (or anti – derivative) of the $\frac{1}{cos^2\theta(1-tan\theta)^2}$

Answer:

$$\frac{1}{cos^2\theta(1-tan\theta)^2} = \frac{sec^2\theta}{(1-tan\theta)^2}$$

Suppose,

$$(1-tan\theta) = z$$
$$sec^2\theta d\theta = dz$$
$$\int \frac{sec^2\theta}{(1-tan\theta)^2}d\theta = \int -\frac{dz}{z^2}$$
$$=-\int z^{-2}dz$$
$$= \frac{1}{z} + C$$
$$= \frac{1}{1-tan\theta} + C$$

Question 26:

Obtain an integral (or anti – derivative) of the $\frac{cos\sqrt{\theta}}{\sqrt{\theta}}$

Answer:

Suppose, $\sqrt{\theta} = z$

$$\frac{1}{2\sqrt{\theta}}d\theta = dz$$
$$\int \frac{cos\sqrt{\theta}}{\sqrt{\theta}} = 2\int cos\ zdz$$
$$= 2sin\ z + C$$
$$= 2sin\ \sqrt{\theta} + C$$

Question 27:

Obtain an integral (or anti – derivative) of the $\sqrt{sin\ 2\theta}\ cos\ 2\theta$

Answer:

Suppose, $sin\ 2\theta = z$

$$2cos\ 2\theta d\theta = dz$$
$$\int \sqrt{sin\ 2\theta}\ cos\ 2\theta = \frac{1}{2}\int \sqrt{z}dz$$
$$= \frac{1}{2}\left(\frac{z^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{1}{3}z^{\frac{3}{2}} + C$$
$$= \frac{1}{3}(sin\ 2\theta)^{\frac{3}{2}} + C$$

Question 28:

Obtain an integral (or anti – derivative) of the $\frac{cos\ \theta}{\sqrt{1+sin\ \theta}}$

Answer:

Suppose, $1+sin\ \theta=z$

$cos\ \theta d\theta=dz$

$\int\frac{cos\ \theta}{\sqrt{1+sin\ \theta}}d\theta=\int\frac{dz}{\sqrt{z}}$

$=\frac{z^{\frac{1}{2}}}{\frac{1}{2}}+C$

$=2\sqrt{z}+C$

$=2\sqrt{1+sin\ \theta}+C$

Question 29:

Obtain an integral (or anti – derivative) of the $cot\ \theta\ log\ sin\ \theta$

Answer:

Suppose, $log\ sin\ \theta=z$

$\frac{1}{sin\ \theta}.\ cos\ \theta=dz$

$cot\ \theta\ d\theta=dz$

$\int cot\ \theta\ log\ sin\ \theta d\theta=\int z\ dz$

$=\frac{z^2}{2}+C$

$=\frac{1}{2}(log\ sin\ \theta)^2+C$

Question 30:

Obtain an integral (or anti – derivative) of the $\frac{sin\ \theta}{1+cos\ \theta}$

Answer:

Suppose,

$1+cos\ \theta=z – sin\ \theta d\theta=dz$

$\int\frac{sin\ \theta}{1+cos\ \theta}d\theta=\int –\frac{dz}{z}$

$= –\int\frac{dz}{z}dz$

$= –log\,|z|+C$

$= –log\,|1+cos\ \theta|+C$

Question 31:

Obtain an integral (or anti – derivative) of the $\frac{sin\ \theta}{(1+cos\ \theta)^2}$

Answer:

Suppose, $1+cos\ \theta=z – sin\ \theta d\theta=dz$

$\int\frac{sin\ \theta}{1+cos\ \theta}d\theta=\int –\frac{dz}{z^2}$

$= –\int\frac{dz}{z^2}dz$

$= –\int z^{-2}dz$

$=\frac{1}{z}+C$

$=\frac{1}{1+cos\ \theta}+C$

Question 32:

Obtain an integral (or anti – derivative) of the $\frac{1}{1+cot\ \theta}$

Answer:

Suppose, I = $\int\frac{1}{1+cot\ \theta}d\theta$

$=\int\frac{1}{1+\frac{cos\ \theta}{sin\ \theta}}d\theta$

$=\int\frac{sin\ \theta}{sin\ \theta+cos\ \theta}d\theta$

$=\frac{1}{2}\int\frac{2sin\ \theta}{sin\ \theta+cos\ \theta}d\theta$

$=\frac{1}{2}\int\frac{(sin\ \theta+cos\ \theta)+(sin\ \theta – cos\ \theta)}{(sin\ \theta+cos\ \theta)}d\theta$

$=\frac{1}{2}\int 1d\theta+\frac{1}{2}\int\frac{(sin\ \theta – cos\ \theta)}{(sin\ \theta+cos\ \theta)}d\theta$

$=\frac{1}{2}\theta+\frac{1}{2}\int\frac{(sin\ \theta – cos\ \theta)}{(sin\ \theta+cos\ \theta)}d\theta$

$Suppose,\ (sin\ \theta+cos\ \theta)=z$

$=(cos\ \theta – sin\ \theta)d\theta=dz$

$I=\frac{\theta}{2}+\frac{1}{2}log\,|z|+C$

$= \frac{\theta}{2} - \frac{1}{2} log\,|(sin\ \theta + cos\ \theta)| + C$

Question 33:

Obtain an integral (or anti – derivative) of the $\frac{1}{1 - tan\theta}$

Answer:

Suppose,

$$\int \frac{1}{1 - tan\ \theta} d\theta$$
$$= \int \frac{1}{1 - \frac{sin\ \theta}{cos\ \theta}} d\theta$$
$$= \int \frac{cos\ \theta}{cos\ \theta - sin\ \theta} d\theta$$
$$= \frac{1}{2} \int \frac{2cos\ \theta}{cos\ \theta - sin\ \theta} d\theta$$
$$= \frac{1}{2} \int \frac{(cos\ \theta - sin\ \theta) + (cos\ \theta + sin\ \theta)}{(cos\ \theta - sin\ \theta)} d\theta$$
$$= \frac{1}{2} \int 1 d\theta + \frac{1}{2} \int \frac{(cos\ \theta + sin\ \theta)}{(cos\ \theta - sin\ \theta)} d\theta$$
$$= \frac{1}{2}\theta + \frac{1}{2} \int \frac{(cos\ \theta + sin\ \theta)}{(cos\ \theta - sin\ \theta)} d\theta$$

$$Suppose,\ (cos\ \theta - sin\ \theta) = z$$
$$= (-sin\ \theta - cos\ \theta) d\theta = dz$$
$$I = \frac{\theta}{2} - \frac{1}{2} log\,|z| + C$$
$$= \frac{\theta}{2} - \frac{1}{2} log\,|(cos\ \theta - sin\ \theta)| + C$$

Question 34:

Obtain an integral (or anti – derivative) of the $\frac{\sqrt{tan\theta}}{sin\ \theta\ cos\ \theta}$

Answer:

Suppose, $Suppose, I = \frac{\sqrt{tan\theta}}{sin\ \theta\ cos\ \theta} d\theta$

$$= \frac{\sqrt{tan\theta} \times cos\ \theta}{sin\ \theta\ cos\ \theta \times cos\ \theta} d\theta$$
$$= \int \frac{\sqrt{tan\theta}}{tan\ \theta\ cos^2\theta} d\theta$$
$$= \int \frac{sec^2\theta}{\sqrt{tan\ \theta}} d\theta$$
$$Suppose, tan\ \theta = z$$
$$sec^2\theta d\theta = dz$$
$$I = \int \frac{dz}{\sqrt{z}}$$
$$= 2\sqrt{z} + C$$
$$= 2\sqrt{tan\ \theta} + C$$

Question 35:

Obtain an integral (or anti – derivative) of the $\frac{(1 + logu)^2}{u}$

Answer:

Suppose, $Suppose, 1 + logu = z$

$$\frac{1}{u} du = dz$$
$$\int \frac{(1 + logu)^2}{u} du = \int z^2\ dz$$
$$= \frac{z^3}{3} + C$$
$$= \frac{(1 + logu)^3}{3} + C$$

Question 36:

Obtain an integral (or anti – derivative) of the $\frac{(u+1)(u + logu)^2}{u}$

Answer:

$$\frac{(u+1)(u + logu)^2}{u} = \frac{(u+1)}{u}(u + logu)^2 = (1 + \frac{1}{u})(u + logu)^2$$
$$Suppose,\ (u + logu) = z$$
$$(1 + \frac{1}{u}) du = dz$$
$$\int (1 + \frac{1}{u})(u + log\ u)^2 du = \int z^2 dz$$
$$= \frac{z^3}{3} + C$$
$$= \frac{1}{3}(u + logu)^3 + C$$

Question 37:

Obtain an integral (or anti – derivative) of the $\frac{u^3\ sin\ (tan^{-1}u^4)}{1 + u^8}$

Answer:

$$Suppose,\ u^4 = z$$
$$4u^3 du = dz$$
$\int u^3\ sin\ (tan^{-1}u^4)$ ... $1 \int sin\ (tan^{-1}z)$ ... (1)

$\int \frac{\quad}{1+u^8} du = \frac{\quad}{4} \int \frac{\quad}{1+z^2}\ dz \ldots . (1)$

$Suppose,\ tan^{-1}z = s$

$\frac{1}{1+z^2}dz = ds$

$From\ (1),\ we\ get$

$\int \frac{u^3\ sin\ (tan^{-1}u^4)}{1+u^8}du = \frac{1}{4}\int sin\ s\ ds$

$= \frac{1}{4}(-coss) + c$

$=- \frac{1}{4}cos(tan^{-1}z) + C$

$=- \frac{1}{4}cos(tan^{-1}u^4) + C$

Question 38:

Which of the following below is the answer for $\int \frac{10u^9+10^u log_e 10}{u^{10}+10^u}du$:

$(a)10^u - u^{10} + C$

$(b)10^u + u^{10} + C$

$(c)(10^u - u^{10})^{-1} + C$

$(d)log(10^u + u^{10}) + C$

Answer:

$u^{10} + 10^u = z$

$(10u^9 + 10^u log_e 10)du = dz$

$\int \frac{10u^9+10^u log_e 10}{u^{10}+10^u}du = \int \frac{dz}{z}$

$= logz + C$

$= log(u^{10} + 10^u) + C$

$Therefore,\ D\ is\ the\ correct\ answer$

Question 39:

Which of the following below is the answer for $\int \frac{du}{sin^2\ u\ cos^2\ u}$

$(a)tan\ u + cot\ u + C$

$(b)tan\ u - cot\ u + C$

$(c)tan\ u\ cot\ u + C$

$(d)tan\ u - cot\ 2u + C$

Answer:

$I = \int \frac{du}{sin^2\ u\ cos^2\ u}$

$= \int \frac{1}{sin^2\ u\ cos^2\ u}du$

$= \int \frac{sin^2\ u\ + cos^2\ u}{sin^2\ u\ cos^2\ u}du$

$= \int \frac{sin^2\ u}{sin^2\ u\ cos^2\ u}du + \int \frac{cos^2\ u}{sin^2\ u\ cos^2\ u}du$

$= \int sec^2\ udu + \int cosec^2\ udu$

$= tan\ u - cot\ u + C$

Therefore, B is the correct answer

Exercise 7.3

Question 1:

Obtain an integral (or anti – derivative) of the sin ^{2} (2u + 5)

Answer 1:

$sin^2(2u + 5) = \frac{1 - cos\ 2(2u+5)}{2} = \frac{1 - cos\ (4u+10)}{2}$

$\int sin^2(2u + 5)du = \int \frac{1 - cos\ (4u+10)}{2}du$

$= \frac{1}{2}\int 1du - \frac{1}{2}\int cos\ (4u + 10)du$

$= \frac{1}{2}u - \frac{1}{2}\frac{sin(4u+10)}{4} + C$

$= \frac{1}{2}u - \frac{1}{8}[sin(4u + 10)] + C$

Question 2:

Obtain an integral (or anti – derivative) of the sin 3u. cos 4u

Answer 2:

As we know, $sin\ C cos\ D = \frac{1}{2}[sin\ (C+D) + sin\ (C – D)]$

$$\int sin\ 3u.\ cos\ 4u\ du = \int \frac{1}{2}[sin\ (3u+4u) + sin\ (3u – 4u)]$$
$$= \int \frac{1}{2}[sin\ (7u) + sin\ (-u)]du$$
$$= \int \frac{1}{2}[sin\ (7u) – sin\ (u)]du$$
$$= \frac{1}{2}\int sin\ (7u)\ du – \frac{1}{2}\int sin\ (u)\ du$$
$$= \frac{1}{2}(\frac{-cos\ 7u}{7}) – \frac{1}{2}(-cos\ u) + C$$
$$= \frac{-cos\ 7u}{14} + \frac{cos\ u}{2} + C$$

Question 3:

Obtain an integral (or anti – derivative) of the cos 2u cos 4u cos 6u

Answer 3:

As we know, $cos\ C\ cos\ D = \frac{1}{2}[cos\ (C+D) + cos\ (C – D)]$

$$\int cos\ 2u\ (cos\ 4u\ cos\ 6u)du = \int cos\ 2u\ [\frac{1}{2}(cos\ (4u+6u) + cos\ (4u – 6u))]du$$
$$\frac{1}{2}\int [cos\ 2u\ (cos\ (10u) + cos\ (-2u))]du$$
$$\frac{1}{2}\int [cos\ 2u\ cos\ 10u + cos\ 2u\ cos\ (-2u)]du$$
$$\frac{1}{2}\int [cos\ 2u\ cos\ 10u + cos^2\ 2u]\ du$$
$$\frac{1}{2}\int [\frac{1}{2}\ (cos\ (2u+10u) + cos\ (2u – 10u)) + (\frac{1+cos\ 4u}{2})]\ du$$
$$\frac{1}{4}\int (cos\ 12u + cos\ 8u + 1 + cos\ 4u)\ du$$
$$\frac{1}{4}[\frac{sin12u}{12} + \frac{sin8u}{8} + u + \frac{sin4u}{4}] + C$$

Question 4:

Obtain an integral (or anti – derivative) of the $sin^3(2u+1)$

Answer 4:

$$I = \int sin^3(2u+1)du$$
$$\int sin^3(2u+1)du = \int sin^2\ (2u+1).\ sin\ (2u+1)\ du$$
$$Supose,\ cos\ (2u+1) = z$$
$$-2sin(2u+1)\ du = dz$$
$$sin(2u+1)\ du = \frac{-dz}{2}$$
$$I = \frac{-1}{2}\int (1 – z^2)dz$$
$$= \frac{-1}{2}\left\{z – \frac{z^3}{3}\right\} + C$$
$$= \frac{-1}{2}\left\{cos\ (2u+1) – \frac{cos^3\ (2u+1)}{3}\right\} + C$$
$$= \frac{-cos\ (2u+1)}{2} + \frac{cos^3\ (2u+1)}{6} + C$$

Question 5:

Obtain an integral (or anti – derivative) of the $sin^3\ u\ cos^3\ u$

Answer 5:

$$I = \int sin^3\ u\ cos^3\ u\ du$$
$$= \int cos^3\ u\ .\ sin^2\ u\ sin\ u\ du$$
$$= \int cos^3\ u\ (1 – cos^2\ u).\ sin\ u\ du$$
$$Suppose,\ cos\ u = z$$
$$-sin\ u\ du = dz$$
$$I = – \int z^3(1 – z^2)dz$$
$$= – \int (z^3 – z^5)dz$$
$$= – \left\{\frac{z^4}{4} – \frac{z^6}{6}\right\} + C$$
$$= – \left\{\frac{cos^4}{4} – \frac{cos^6}{6}\right\} + C$$
$$= \frac{cos^6}{6} – \frac{cos^4}{4} + C$$

Question 6:

Obtain an integral (or anti – derivative) of the sin u sin 2u sin 3u

Answer 6:

$$sin\ C\ sin\ D = \frac{1}{2}[cos\ (C – D) – cos\ (C+D)]$$
$$\int sin\ u\ sin\ 2u\ sin\ 3u\ du = \int sin\ u\ .\ \frac{1}{2}\{cos\ (2u – 3u) – cos\ (2u+3u)\}\ du$$
$$= \frac{1}{4}\left[\frac{-cos\ 2u}{2}\right] – \frac{1}{2}\int \left\{\frac{1}{2}(sin\ (u+5u) + sin\ (u – 5u))\right\}du$$
$$= \frac{-cos\ 2u}{8} – \frac{1}{4}\int \{(sin\ (6u) + sin\ (-4u))\}\ du$$

$= \frac{1}{2}\int(sin\ u\ cos\ (-u) - sin\ u\ cos\ 5u)\ du$

$= \frac{1}{2}\int(sin\ u\ cos\ u - sin\ u\ cos\ 5u)\ du$

$= \frac{1}{2}\int\frac{(2sin\ u\ cos\ u)}{2}\ du - \frac{1}{2}\int sin\ u\ cos\ 5u\ du$

$= \frac{1}{2}\int\frac{(sin\ 2u)}{2}\ du - \frac{1}{2}\int sin\ u\ cos\ 5u\ du$

$= \frac{-cos\ 2u}{8} - \frac{1}{4}\left[\frac{-cos\ 6u}{6} + \frac{cos\ 4u}{4}\right] + C$

$= \frac{-cos\ 2u}{8} - \frac{1}{8}\left[\frac{-cos\ 6u}{3} + \frac{cos\ 4u}{2}\right] + C$

$= \frac{1}{8}\left[\frac{-cos\ 6u}{3} - \frac{cos\ 4u}{2} - cos\ 2u\right] + C$

Question 7:

Obtain an integral (or anti – derivative) of the sin 4u sin 8u

Answer 7:

As we know, $sin\ C\ sin\ D = \frac{1}{2}[cos\ (C-D) - cos\ (C+D)]$

$\int sin\ 4u\ sin\ 8u\ du = \int\frac{1}{2}[cos\ (4u-8u) - cos\ (4u+8u)]\ du$

$= \frac{1}{2}\int(cos\ (-4u) - cos\ 12u)\ du$

$= \frac{1}{2}\int(cos\ 4u - cos\ 12u)\ du$

$= \frac{1}{2}\left[\frac{sin\ 4u}{4} - \frac{sin\ 12u}{12}\right]$

Question 8:

Obtain an integral (or anti – derivative) of the $\frac{1-cos\ u}{1+cos\ u}$

Answer 8:

$\frac{1-cos\ u}{1+cos\ u} = \frac{2sin^2\ \frac{u}{2}}{2cos^2\ \frac{u}{2}}$

$= tan^2\ \frac{u}{2}$

$= (sec^2\ \frac{u}{2} - 1)$

$\int\frac{1-cos\ u}{1+cos\ u}du = \int(sec^2\ \frac{u}{2} - 1)\ du$

$= \left[\frac{tan\ \frac{u}{2}}{\frac{1}{2}} - u\right] + C$

$= 2tan\ \frac{u}{2} - u + C$

Question 10:

Obtain an integral (or anti – derivative) of the sin ^{4} u.

Answer 10:

$sin^4\ u = sin^2\ u \times sin^2\ u$

$= (\frac{1-cos\ 2u}{2})(\frac{1-cos\ 2u}{2})$

$= \frac{1}{4}(1-cos\ 2u)^2$

$= \frac{1}{4}(1+cos^2\ 2u - 2\ cos\ 2u)$

$= \frac{1}{4}\left[1 + (\frac{1+cos\ 4u}{2}) - 2cos\ 2u\right]$

$= \frac{1}{4}[1 + \frac{1}{2} + \frac{1}{2}\ cos\ 4u - 2cos\ 2u]$

$= \frac{1}{4}[\frac{3}{2} + \frac{1}{2}\ cos\ 4u - 2cos\ 2u]$

$\int sin^4\ u\ du = \frac{1}{4}\ \int[\frac{3}{2} + \frac{1}{2}\ cos\ 4u - 2cos\ 2u]\ du$

$= \frac{1}{4}\ [\frac{3}{2}u + \frac{1}{2}\ (\frac{sin\ 4u}{4}) - sin\ 2u] + C$

$= \frac{3}{8}u + (\frac{sin\ 4u}{32}) - \frac{sin\ 2u}{4} + C$

Question 11:

Obtain an integral (or anti – derivative) of the cos ^{4} 2u

Answer 11:

$cos^4\ 2u = (sin^2\ 2u)^2$

$= (\frac{1+cos\ 4u}{2})^2$

$= \frac{1}{4}(1+cos^2\ 4u + 2\ cos\ 4u)$

$= \frac{1}{4}\left[1 + (\frac{1+cos\ 8u}{2}) + 2cos\ 4u\right]$

$= \frac{1}{4}[1 + \frac{1}{2} + \frac{1}{2}\ cos\ 8u + 2cos\ 4u]$

$= \frac{1}{4}[\frac{3}{2} + \frac{1}{2}\ cos\ 8u + 2cos\ 4u]$

$\int cos^4\ u\ du = \frac{1}{4}\ \int[\frac{3}{2} + \frac{1}{2}\ cos\ 8u + 2cos\ 4u]\ du$

$= \frac{1}{4}\ [\frac{3}{2}u + \frac{1}{2}\ (\frac{sin\ 8u}{4}) + sin\ 4u] + C$

$= \frac{3}{8}u + (\frac{sin\ 8u}{64}) + \frac{sin\ 4u}{8} + C$

Question 12:

Obtain an integral (or anti – derivative) of the $\frac{sin^2\ u}{1+cos\ u}$

Answer 12:

$\frac{sin^2\ u}{1+cos\ u} = \frac{(2sin\ \frac{u}{2}\ cos\ \frac{u}{2})^2}{2\ cos^2\frac{u}{2}}$ $[Since\ sin\ u = 2\ sin\ \frac{u}{2}\ cos\ \frac{u}{2};\ cos\ u = 2\ cos^2\frac{u}{2} - 1]$

$= \frac{4sin^2\ \frac{u}{2}\ cos^2\ \frac{u}{2}}{2\ cos^2\frac{u}{2}}$

$= 2sin^2\ \frac{u}{}$

$= 2sin^2 \frac{u}{2}$

$= 1 - cos\ u$

$\int \frac{sin^2\ u}{1+cos\ u} du = \int (1 - cos\ u)\ du$

$= u - sin\ u + C$

Question 13:

Obtain an integral (or anti – derivative) of the $\frac{cos\ 2u - cos\ 2a}{cos\ u - cos\ a}$

Answer 13:

$\frac{cos\ 2u - cos\ 2a}{cos\ u - cos\ a} = \frac{-2\ sin\ \frac{2u+2a}{2}\ sin\ \frac{2u-2a}{2}}{-2\ sin\ \frac{u+a}{2}\ sin\ \frac{u-a}{2}}$ $\left[Since,\ cos\ A - cos\ B = -2\ sin\ \frac{A+B}{2}\ sin\ \frac{A-B}{2}\right]$

$= \frac{sin\ (u+a)\ sin\ (u-a)}{sin\ \frac{u+a}{2}\ sin\ \frac{u-a}{2}}$

$= \frac{\left[2sin\ \frac{u+a}{2}\ cos\ \frac{u+a}{2}\right]\left[2sin\ \frac{u-a}{2}\ cos\ \frac{u-a}{2}\right]}{sin\ \frac{u+a}{2}\ sin\ \frac{u-a}{2}}$

$= 4cos\ \frac{u+a}{2}\ cos\ \frac{u-a}{2}$

$= 2\left[cos\ \left(\frac{u+a}{2} + \frac{u-a}{2}\right) + cos\ \left(\frac{u+a}{2} - \frac{u-a}{2}\right)\right]$

$= 2[cos\ (u) + cos\ a]$

$= 2cos\ u + 2cos\ a$

Question 14:

Obtain an integral (or anti – derivative) of the $\frac{cos\ u - sin\ u}{1+sin\ 2u}$

Answer 14:

$\frac{cos\ u - sin\ u}{1+sin\ 2u} = \frac{cos\ u - sin\ u}{(sin^2\ u + cos^2\ u) + 2sin\ u\ cos\ u}$

$Since,\ sin^2\ u + cos^2\ u = 1; sin\ 2u = 2sin\ u\ cos\ u$

$= \frac{cos\ u - sin\ u}{(sin\ u + cos\ u)^2}$

$Suppose,\ sin\ u + cos\ u = z$

$(cos\ u - sin\ u)\ du = dz$

$\int \frac{cos\ u - sin\ u}{1+sin\ 2u}\ du = \int \frac{cos\ u - sin\ u}{(sin\ u + cos\ u)^2} du$

$= \frac{dz}{z^2}$

$= \int z^{-2} dz$

$= -z^{-1} + C$

$= -\frac{1}{z} + C$

$= \frac{-1}{sin\ u + cos\ u} + C$

Question 15:

Obtain an integral (or anti – derivative) of the $tan^3\ 2u\ sec\ 2u$

Answer 15:

$tan^3\ 2u\ sec\ 2u = tan^2\ 2u\ tan\ 2u\ sec\ 2u$

$= (sec^2\ 2u - 1) tan\ 2u\ sec\ 2u$

$= sec^2\ 2u\ tan\ 2u\ sec\ 2u - tan\ 2u\ sec\ 2u$

$= \int tan^3\ 2u\ sec\ 2udu = \int sec^2\ 2u\ tan\ 2u\ sec\ 2u\ du - \int tan\ 2u\ sec\ 2u\ du$

$= \int sec^2\ 2u\ tan\ 2u\ sec\ 2u\ du - \frac{sec\ 2u}{2} + C$

$Suppose,\ sec\ 2u = z$

$2sec\ 2u\ tan\ 2udu = dz$

$\int tan^3\ 2u\ sec\ 2u = \frac{1}{2} z^2 dz - \frac{sec\ 2u}{2} + C$

$= \frac{z^3}{6} - \frac{sec\ 2u}{2} + C$

$= \frac{(sec\ 2u)^3}{6} - \frac{sec\ 2u}{2} + C$

Question 16:

Obtain an integral (or anti – derivative) of the $tan^4\ u$

Answer 16:

$tan^4\ u = tan^2\ u\ \times tan^2\ u$

$= (sec^2\ u - 1) tan^2\ u$

$= sec^2\ u\ tan^2\ u - tan^2\ u$

$= sec^2\ u\ tan^2\ u - (sec^2\ u - 1)$

$= sec^2\ u\ tan^2\ u - sec^2\ u\ + 1$

$\int tan^4\ u\ du = \int sec^2\ u\ tan^2\ u\ du - \int sec^2\ u\ du + \int 1\ du$

$= \int sec^2\ u\ tan^2\ u\ du - tan\ u + u + C\ \ \ldots\ldots(1)$

$Now,\ \int sec^2\ u\ tan^2\ u\ du$

$Suppose,\ tan\ u = z$

$sec^2\ u\ du = dz$

$\int sec^2\ u\ tan^2\ u\ du = \int z^2\ dz$

$= \frac{z^3}{3} + C$

$= \frac{tan^3\ u}{3}$

$Therefore, from equation (1) is$

$\int tan^4\ u\ du = \frac{tan^3\ u}{3} - tan\ u + u + C$

Question 17:

Obtain an integral (or anti – derivative) of the $\frac{sin^3\ u + cos^3\ u}{sin^2\ u\ cos^2\ u}$

Answer 17:

$\frac{sin^3\ u + cos^3\ u}{sin^2\ u\ cos^2\ u} = \frac{sin^3\ u}{sin^2\ u\ cos^2\ u} + \frac{cos^3\ u}{sin^2\ u\ cos^2\ u} = \frac{sin\ u}{cos^2\ u} + \frac{cos\ u}{sin^2\ u} = tan\ u\ sec\ u + cot\ u\ cosec\ u$

$Therefore,\ \int \frac{sin^3\ u + cos^3\ u}{sin^2\ u\ cos^2\ u}\ du = \int (tan\ u\ sec\ u + cot\ u\ cosec\ u)\ du$

$= sec\ u - cosec\ u + C$

Question 18:

Obtain an integral (or anti – derivative) of the $\frac{cos\ 2u + 2sin^2\ u}{cos^2\ u}$

Answer 18:

$\frac{\cos\ 2u+2sin^2\ u}{cos^2\ u}$

$\frac{\cos\ 2u+(1-\cos\ 2u)}{cos^2\ u}\ \ [Since,\ \cos\ 2u=1-2sin^2\ u]$

$=\frac{1}{cos^2\ u}$

$=sec^2\ u$

$\int\frac{\cos\ 2u+2sin^2\ u}{cos^2\ u}\ du=\int sec^2\ u\ du=\tan\ u+C$

Question 19:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sin\ u\ cos^2\ u}$

Answer 19:

$\frac{1}{\sin\ u\ cos^2\ u}=\frac{sin^2\ u+cos^2\ u}{\sin\ u\ cos^2\ u}$

$=\frac{\sin\ u}{cos^2\ u}+\frac{1}{\sin\ u\ \cos\ u}$

$=\tan\ u\ sec^2\ u+\frac{1}{\sin\ u\ \cos\ u}\times\frac{cos^2\ u}{cos^2\ u}$

$=\tan\ u\ sec^2\ u+\frac{sec^2\ u}{\tan\ u}$

$\int\frac{1}{\sin\ u\ cos^2\ u}du=\int\tan\ u\ sec^2\ u\ du+\int\frac{sec^2\ u}{\tan\ u}\ du$ $Suppose,\ \tan\ u=z$

$sec^2\ u\ du=dz$

$\int\frac{1}{\sin\ u\ cos^2\ u}\ du=\int z\ dz+\int\frac{1}{z}\ dz$

$=\frac{z^2}{2}+log\,|z|+C$

$=\frac{1}{2}tan^2\ u+log\,|\tan\ u|+C$

Question 20:

Obtain an integral (or anti – derivative) of the $\frac{\cos\ 2u}{(\cos\ u+\sin\ u)^2}$

Answer 20:

$\frac{\cos\ 2u}{(\cos\ u+\sin\ u)^2}=\frac{\cos\ 2u}{cos^2\ u+sin^2\ u+2\sin\ u\ \cos\ u}=\frac{\cos\ 2u}{1+\sin\ 2u}$

$\int\frac{\cos\ 2u}{(\cos\ u+\sin\ u)^2}\ du=\int\frac{\cos\ 2u}{(1+\sin\ 2u)}\ du$

$Suppose,\ 1+\sin\ 2u=Z$

$2\cos\ 2u\ du=dz$

$\int\frac{\cos\ 2u}{(\cos\ u+\sin\ u)^2}\ du=\frac{1}{2}\int\frac{1}{z}\ dz$

$=\frac{1}{2}log\,|z|+C$

$=\frac{1}{2}log\,|1+\sin\ 2u|+C$

$=\frac{1}{2}log\left|(\cos\ u+\sin\ u)^2\right|+C$

$=\frac{1}{2}log\,|\cos\ u+\sin\ u|+C$

Question 21:

Obtain an integral (or anti – derivative) of the $sin^{-1}(\cos\ u)$

Answer 21:

$sin^{-1}(\cos\ u)$

$Suppose,\ \cos\ x=z$

$Then,\ \sin\ u=\sqrt{1-u^2}$

$(-\sin\ u)\ du=dz$

$du=\frac{-dz}{\sqrt{1-z^2}}$

$Therefore,\ \int sin^{-1}(\cos\ u)\ du=\int sin^{-1}\ z(-\frac{dz}{\sqrt{\ }})\ dz$

$Therefore,\ \int sin^{-1}(\cos\ u)\ du=-\int p\ dp$

$=-\frac{p^2}{2}+C$

$=-\frac{(sin^{-1}\ z)^2}{2}+C$

$=-\frac{(sin^{-1}\ (\cos\ u))^2}{2}+C\ .....(1)$

$As\ we\ know,$

$\int sin^{-1}(\cos\ u)\ du=\frac{-\left[\frac{\pi}{2}-u\right]^2}{2}+C$

$=-\frac{1}{2}\left((\frac{\pi}{2})^2+u^2-\pi u\right)+C$

$=-\frac{(\pi)^2}{8}-\frac{u^2}{2}+\frac{1}{2}\pi u+C$

$=\frac{\pi u}{2}-\frac{u^2}{2}+\left(c-\frac{(\pi)^2}{8}\right)=\frac{\pi u}{2}-\frac{u^2}{2}+C_1$

$$Suppose,\ sin^{-1} z = p$$
$$\frac{1}{\sqrt{1-z^2}}\ dz = dp$$
$$sin^{-1} u + cos^{-1} u\ = \frac{\pi}{2}$$
$$There fore,\ sin^{-1}(cos\ u) = \frac{\pi}{2} - cos^{-1}(cos\ u) = (\frac{\pi}{2} - u)$$
$$On\ substituting\ in\ equation\ (1), we\ get,$$

Question 22:

Obtain an integral (or anti – derivative) of the $\frac{1}{cos\ (u-m)\ cos\ (u-n)}$

Answer 22:

$$\frac{1}{cos\ (u-m)\ cos\ (u-n)} = \frac{1}{sin\ (m-n)}\left[\frac{sin\ (m-n)}{cos\ (u-m)\ cos\ (u-n)}\right]$$
$$= \frac{1}{sin\ (m-n)}\left[\frac{sin\ [(u-n)-(u-m)]}{cos\ (u-m)\ cos\ (u-n)}\right]$$
$$= \frac{1}{sin\ (m-n)}\frac{sin\ (u-n)\ cos\ (u-m) - cos\ (u-n)\ sin\ (u-m)}{cos\ (u-m)\ cos\ (u-n)}$$
$$= \frac{1}{sin\ (m-n)}[tan\ (u-n) - tan\ (u-m)]$$
$$\int \frac{1}{cos\ (u-m)\ cos\ (u-n)} du = \frac{1}{sin\ (m-n)} \int [tan\ (u-n) - tan\ (u-m)] du$$
$$= \frac{1}{sin\ (m-n)}[-log\,|cos\ (u-n)| + log\,|cos\ (u-m)|]$$
$$= \frac{1}{sin\ (m-n)} log\ \left[\frac{cos\ (u-m)}{cos\ (u-n)}\right] + C$$

Question 23:

Which of the following below is the answer for $\frac{sin^2\ u - cos^2\ u}{sin^2\ u\ cos^2\ u}$

(a) tan u + cot u + C

(b) tan u + cosec u + C

(c) – tan u + cot u + C

(d) tan u + sec u + C

Answer 23:

$$\int \frac{sin^2\ u - cos^2\ u}{sin^2\ u\ cos^2\ u}\ du$$
$$= \int \left(\frac{sin^2\ u}{sin^2\ u\ cos^2\ u} - \frac{cos^2\ u}{sin^2\ u\ cos^2\ u}\right)\ du$$
$$= \int (sec^2\ u - cosec^2\ u)\ du$$
$$= tan\ u + cot\ u + C$$

Thus, (a) is the correct answer.

Question 24:

Which of the following below is the answer for $\int \frac{e^u(1+u)}{cos^2\ (e^u u)}\ du$

(a) – cot (e u^{u}) + C

(b) tan (e u^{u}) + C

(c) tan (e^{u}) + C

(d) cot (e^{u}) + C

Answer 24:

$$\int \frac{e^u(1+u)}{cos^2\ (e^u u)}\ du$$
$$Suppose,\ e^u u = z$$
$$(e^u . u + e^u . 1) du = \int \frac{dz}{cos^2\ z}$$
$$= \int sec^2\ z dz$$
$$= tan\ z + C$$
$$= tan(e^u . u) + C$$

Thus, (b) is the correct answer.

Exercise 7.4

Question 1:

Obtain an integral (or anti – derivative) of the $\frac{3u^2}{u^6+1}$

Answer 1:

$$Suppose,\ u^3 = z$$
$$3u^2 du = dz$$

$\int \frac{3u}{u^6+1} du = \int \frac{dz}{z^2+1}$
$= tan^{-1} z + C$
$= tan^{-1} u^3 + C$

Question 2:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{1+4u^2}}$

Answer 2:

$Suppose,\ 2u = z$
$2\ du = dz$
$\int \frac{1}{\sqrt{1+4u^2}}\ du = \frac{1}{2} \int \frac{dz}{\sqrt{1+z^2}}$
$= \frac{1}{2}[log\left|z + \sqrt{1+z^2}\right|] + C$
$= \frac{1}{2}[log\left|2u + \sqrt{1+4u^2}\right|] + C$

Question 3:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{(2-u)^2+1}}$

Answer 3:

$Suppose,\ 2-u = z$
$-du = dz$
$\int \frac{1}{\sqrt{(2-u)^2+1}}\ du = -\int \frac{1}{\sqrt{z^2+1}}\ dz$
$= -[log\left|z + \sqrt{z^2+1}\right|] + C$
$= -[log\left|2-u + \sqrt{(2-u)^2+1}\right|] + C$
$= log\left|\frac{1}{(2-u)+\sqrt{u^2-4u+5}}\right| + C$

Question 4:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{9-25u^2}}$

Answer 4:

Suppose, 5u = z

5 du = dz

$\int \frac{1}{\sqrt{9-25u^2}}\ du = \frac{1}{5} \int \frac{1}{9-z^2}\ dz$
$= \frac{1}{5} \int \frac{1}{\sqrt{3^2-z^2}}\ dz$
$= \frac{1}{5} sin^{-1} \frac{z}{3} + C$
$= \frac{1}{5} sin^{-1} \frac{5u}{3} + C$

Question 5:

Obtain an integral (or anti – derivative) of the $\frac{3u}{1+2u^4}$

Answer 5:

$Suppose,\ \sqrt{2}u^2 = z$
$2\sqrt{2}\ u\ du = dz$
$\int \frac{3u}{1+2u^4}\ du = \frac{3}{2\sqrt{2}} \int \frac{dz}{1+z^2} dz$
$= \frac{3}{2\sqrt{2}}[tan^{-1} z] + C$
$= \frac{3}{2\sqrt{2}}[tan^{-1}(\sqrt{2}u^2)] + C$

Question 6:

Obtain an integral (or anti – derivative) of the $\frac{u^2}{1-u^6}$

Answer 6:

Suppose, u^3 = z

3 u^2 du = dz

$\int \frac{u^2}{1-u^6} du = \frac{1}{3} \int \frac{dz}{1-z^2}$
$= \frac{1}{3}\left[\frac{1}{2} log\left|\frac{1+z}{1-z}\right|\right] + C$
$= \frac{1}{6} log\left|\frac{1+u^3}{1-u^3}\right| + C$

Question 7:

Obtain an integral (or anti – derivative) of the $\frac{u-1}{u^2-1}$

Answer 7:

$$\int \frac{u-1}{\sqrt{u^2-1}} du = \int \frac{u}{\sqrt{u^2-1}}\ du - \frac{1}{\sqrt{u^2-1}}\ du$$
$$For, \int \frac{u}{\sqrt{u^2-1}}\ du$$

$$Suppose\ u^2-1 = z$$
$$2u\ du = dz$$
$$Therefore,\ \int \frac{u}{\sqrt{u^2-1}}\ du = \frac{1}{2}\int \frac{dz}{\sqrt{z}}$$
$$= \frac{1}{2}\int z^{-\frac{1}{2}} dz$$
$$= \frac{1}{2}[2z^{\frac{1}{2}}] + C$$
$$= \sqrt{z} + C$$
$$= \sqrt{u^2-1} + C$$

$$From\ the\ above\ equation\ we\ get$$
$$\int \frac{u-1}{\sqrt{u^2-1}} du = \int \frac{u}{\sqrt{u^2-1}}\ du - \frac{1}{\sqrt{u^2-1}}\ du$$
$$= \sqrt{u^2-1} + C - log\left|u + \sqrt{u^2-1}\right| + C_1$$
$$= \sqrt{u^2-1} - log\left|u + \sqrt{u^2-1}\right| + (C + C_1)$$
$$= \sqrt{u^2-1} - log\left|u + \sqrt{u^2-1}\right| + C_2$$

Question 8:

Obtain an integral (or anti – derivative) of the $\frac{u^2}{\sqrt{u^6+m^6}}$

Answer 8:

Suppose, $u^3 = z$

$3u^2\ du = dz$

$$\int \frac{u^2}{\sqrt{u^6+m^6}}\ du = \frac{1}{3}\int \frac{dz}{\sqrt{z^2+(m^3)^2}}$$
$$= \frac{1}{3} log\left|z + \sqrt{z^2+m^6}\right| + C$$
$$= \frac{1}{3} log\left|u^3 + \sqrt{u^6+m^6}\right| + C$$

Question 9:

Obtain an integral (or anti – derivative) of the $\frac{sec^2\ u}{\sqrt{tan^2\ u+4}}$

Answer 9:

Suppose, tan u = z

$sec^2\ u\ du = dz$

$$\int \frac{sec^2\ u}{\sqrt{tan^2\ u+4}}\ du = \int \frac{dz}{\sqrt{z^2+2^2}}$$
$$= log\left|z + \sqrt{z^2+4}\right| + C$$
$$= log\left|tan\ u + \sqrt{tan^2\ u+4}\right| + C$$

Question 10:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{u^2+2u+2}}$

Answer 10:

$$\int \frac{1}{\sqrt{u^2+u+2}} du = \int \frac{1}{(u+1)^2+(1)^2}$$
$$Suppose,\ u+1 = z$$
$$du = dz => \int \frac{1}{\sqrt{u^2+2u+2}} du = \int \frac{1}{\sqrt{z^2+1}}\ dz$$
$$= log\left|z + \sqrt{z^2+1}\right| + C$$
$$= log\left|(u+1) + \sqrt{(u+1)^2+1}\right| + C$$
$$= log\left|(u+1) + \sqrt{u^2+2u+1}\right| + C$$

Question 11:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{9u^2+6u+2}}$

Answer 11:

$$\int \frac{1}{\sqrt{9u^2+6u+2}} du = \int \frac{1}{(3u+1)^2+(2)^2}$$
$$Suppose,\ 3u+1 = z$$
$$3\ du = dz => \int \frac{1}{\sqrt{9u^2+6u+2}} du = \frac{1}{3}\int \frac{1}{\sqrt{t^2+2^2}}\ dz$$
$$= \frac{1}{3}\left[\frac{1}{2} tan^{-1}(\frac{z}{2})\right] + C$$
$$= \frac{1}{3}\left[\frac{1}{2} tan^{-1}(\frac{3u+1}{2})\right] + C$$

Question 12:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{7-6u-u^2}}$

Answer 12:

$7 - 6u - u^2$ = can also be written as $7 - (u^2 + 6u + 9 - 9)$

Therefore,

7 – (u^2 + 6 u + 9 – 9)

= 16 – (u^2 + 6 u + 9)

= 16 – (u + 3) ^{2}

= 4 ^{2} – (u + 3) ^{2}

$$\int \frac{1}{\sqrt{7-6u-u^2}}du = \int \frac{1}{4^2-(u+3)^2}du$$
$$Suppose,\ u+3=z$$
$$du = dz$$
$$\int \frac{1}{4^2-(u+3)^2}du = \int \frac{1}{4^2-(z)^2}dz$$
$$= sin^{-1}(\tfrac{z}{4})+C$$
$$= sin^{-1}(\tfrac{u+3}{4})+C$$

Question 13:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{(u-1)(u-2)}}$

Answer 13:

(u – 1) (u – 2) can be written as u ^{2} – 3 u + 2

Therefore,

u ^{2} – 3 u + 2

$$= u^2-3u+\tfrac{9}{4}-\tfrac{9}{4}+2\ \int \frac{1}{\sqrt{(u-1)(u-2)}}\ du = \int \frac{1}{\sqrt{\left(u-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}\ du\ \int \frac{1}{\sqrt{\left(u-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}\ du = \int \frac{1}{\sqrt{z^2-(\frac{1}{2})^2}}dz$$
$$= \left(u-\tfrac{3}{2}\right)^2-\tfrac{1}{4}$$
$$= \left(u-\tfrac{3}{2}\right)^2-\left(\tfrac{1}{2}\right)^2 \quad Suppose,\ u-\tfrac{3}{2}=z$$
$$du = dz$$
$$= log\left|z+\sqrt{z^2-(\tfrac{1}{2})^2}\right|+C$$
$$= log\left|(u-\tfrac{3}{2})+\sqrt{(u-\tfrac{3}{2})^2-(\tfrac{1}{2})^2}\right|+C$$
$$= log\left|(u-\tfrac{3}{2})+\sqrt{u^2-3u+2}\right|+C$$

Question 14:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{8+3u-u^2}}$

Answer 14:

$$\frac{1}{\sqrt{8+3u-u^2}} can\ also\ be\ written\ as\ 8-\left(u^2-3u+\tfrac{9}{4}-\tfrac{9}{4}\right)\ Suppose\ u-\tfrac{3}{2}=z$$
$$du = dz$$
$$There fore,\ \tfrac{41}{4}-\left(u-\tfrac{3}{2}\right)^2$$
$$\int \frac{1}{\sqrt{8+3u-u^2}}\ du = \int \frac{1}{\sqrt{\frac{41}{4}-\left(u-\frac{3}{2}\right)^2}}\ du$$
$$\int \frac{1}{\sqrt{\frac{41}{4}-\left(u-\frac{3}{2}\right)^2}}\ du = \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-z^2}}\ dz$$
$$= sin^{-1}\left(\frac{z}{\frac{\sqrt{41}}{2}}\right)+C$$
$$= sin^{-1}\left(\frac{u-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right)+C$$
$$= sin^{-1}\left(\frac{2u-3}{\sqrt{41}}\right)+C$$

Question 15:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{(u-m)(u-n)}}$

Answer 15:

$$(u-m)(u-n)\ can\ also\ be\ written\ as\ u^2-(m+n)u+mn\ Suppose,\ u-\left(\tfrac{(m+n)}{2}\right)=z$$
$$There fore,$$
$$u^2-(m+n)u+mn$$
$$= u^2-(m+n)u+\tfrac{(m+n)^2}{4}-\tfrac{(m+n)^2}{4}+mn$$
$$= \left[u-\tfrac{(m+n)}{2}\right]^2-\tfrac{(m-n)^2}{4}$$
$$\int \frac{1}{\sqrt{(u-m)(u-n)}}\ du = \int \frac{1}{\sqrt{\left\{u-\frac{(m+n)}{2}\right\}^2-\left(\frac{(m-n)}{2}\right)^2}}\ du$$
$$du = dz$$
$$\int \frac{1}{\sqrt{\left\{u-\frac{(m+n)}{2}\right\}^2-\left(\frac{(m-n)}{2}\right)^2}}\ du = \int \frac{1}{\sqrt{z^2-\left(\frac{m-n}{2}\right)^2}}\ dz$$
$$log\left|z+\sqrt{z^2-(\tfrac{m-n}{2})^2}\right|+C$$
$$log\left|\{u-(\tfrac{m+n}{2})\}+\sqrt{(u-m)(u-n)}\right|+C$$

Question 16:

Obtain an integral (or anti – derivative) of the $\frac{4u+1}{\sqrt{2u^2+u-3}}$

Answer 16:

$Suppose,\ 4u+1 = A\frac{d}{dx}(2u^2+u-3)+B....(1)$
$4u+1 = A(4u+1)+B$
$4u+1 = 4Au+A+B$

Equate the coefficients of u and the constants on both the sides, we get

4 A = 4 => A = 1

A + B = 1 => B = 0

From (1), we get

Suppose, 2 u^2 + u – 3 = z

(4 u + 1) du = dz

$\int \frac{4u+1}{\sqrt{2u^2+u-3}}du = \int \frac{1}{\sqrt{z}}\ dz$
$= 2\sqrt{z}+C$
$= 2\sqrt{2u^2+u-3}+C$

Question 17:

Obtain an integral (or anti – derivative) of the $\frac{u+2}{\sqrt{u^2-1}}$

Answer 17:

$Suppose,\ u+2 = A\frac{d}{du}(u^2-1)+B....(1)$
$u+2 = A(2u)+B$

Equate the coefficients of u and the constants on both the sides, we get

2 A = 1 => A = (1 / 2)

B = 2

From (1), we get

$From\ (1), we\ get,$
$(u+2) = \frac{1}{2}(2u)+2$
$Then,\ \int \frac{u+2}{\sqrt{u^2-1}}\ du = \int \frac{\frac{1}{2}(2u)+2}{\sqrt{u^2-1}}$
$= \frac{1}{2}\int \frac{2u}{\sqrt{u^2-1}}\ du + \int \frac{2}{\sqrt{u^2-1}}\ du.....(2)$

$In\ \frac{1}{2}\int \frac{2u}{\sqrt{u^2-1}}\ du = \frac{1}{2}\int \frac{dz}{\sqrt{z}}$
$= \frac{1}{2}[2\sqrt{z}]+C$
$= \sqrt{z}+C$
$= \sqrt{u^2-1}$
$Then,\ \int \frac{2}{\sqrt{u^2-1}}\ du = 2\int \frac{1}{\sqrt{u^2-1}}\ du$
$In\ equation\ (2), we\ get,$
$\int \frac{u+2}{\sqrt{u^2-1}}\ du = \sqrt{u^2-1}+2\ log\ \left|u+\sqrt{u^2-1}\right|+C$

Question 18:

Obtain an integral (or anti – derivative) of the $\frac{5u-2}{1+2u+3u^2}$

Answer 18:

Suppose, $5u-2 = A\frac{d}{du}(1+2u+3u^2)+B$
$5u-2 = A(2+6u)+B$

Equate the coefficients of u and the constants on both the sides, we get

$5 = 6A => A = \frac{5}{6}$
$2A+B = -2 => B = -\frac{11}{3}$
$5u-2 = \frac{5}{6}(2+6u)+\frac{-11}{3}$
$\int \frac{5u-2}{1+2u+3u^2}\ du = \int \frac{\frac{5}{6}(2+6u)-\frac{11}{3}}{1+2u+3u^2}\ du$
$= \frac{5}{6}\int \frac{2+6u}{1+2u+3u^2}\ du - \frac{11}{3}\int \frac{1}{1+2u+3u^2}\ du$

$Suppose,\ I_1 = \int \frac{2+6u}{1+2u+3u^2}\ du\ and\ I_2 = \int \frac{1}{1+2u+3u^2}\ du$
$\int \frac{5u-2}{1+2u+3u^2}\ du = \frac{5}{6}I_1 - \frac{11}{3}I_2....(1)$
$I_1 = \int \frac{2+6u}{1+2u+3u^2}\ du\ and\ I_2 = \int \frac{1}{1+2u+3u^2}\ du$
$Suppose,\ 1+2u+3u^2 = z$
$(2+6u)\ du = dz$
$I_1 = \int \frac{dz}{z}$
$I_1 = log\,|z|+C$
$I_1 = log\,|1+2u+3u^2|+C.....(2) I_2 = \int \frac{1}{1+2u+3u^2}\ du$

$1+2u+3u^2 can\ also\ be\ written\ as\ 1+3\left(u^2+\frac{2}{3}\ u\right)$
$Therefore,$
$1+3\left(u^2+\frac{2}{3}\ u\right)$
$= 1+3\left(u^2+\frac{2}{3}\ u+\frac{1}{9}-\frac{1}{9}\right)$
$= 1+3\left(u+\frac{1}{3}\right)^2-\frac{1}{3}$
$= \frac{2}{3}+3\left(u+\frac{1}{3}\right)^2$
$= 3[\left(u+\frac{1}{3}\right)^2+\frac{2}{9}]$
$= 3[\left(u+\frac{1}{3}\right)^2+(\frac{\sqrt{2}}{3})^2]$

$I_2 = \frac{1}{3}\int \left[\frac{1}{[\left(u+\frac{1}{3}\right)^2+(\frac{\sqrt{2}}{3})^2]}\right] du$
$= \frac{1}{3}\left[\frac{1}{\frac{\sqrt{2}}{3}}\ tan^{-1}(\frac{u+\frac{1}{3}}{\frac{\sqrt{2}}{3}})\right]$
$= \frac{1}{3}\left[\frac{3}{\sqrt{2}}tan^{-1}(\frac{3u+1}{\sqrt{2}})\right]+C$
$= \frac{1}{\sqrt{2}}tan^{-1}(\frac{3u+1}{\sqrt{2}})+C.....(3)$

Substituting equations (2) and (3) in equation (1), we get,

$$\int \frac{5u-2}{1+2u+3u^2}\,du = \frac{5}{6}\int \frac{2+6u}{1+2u+3u^2}\,du - \frac{11}{3}\int \frac{1}{1+2u+3u^2}\,du$$
$$\int \frac{5u-2}{1+2u+3u^2}\,du = \frac{5}{6}\left[\log\left|1+2u+3u^2\right|\right] - \frac{11}{3}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3u+1}{\sqrt{2}}\right)\right] + C$$
$$= \frac{5}{6}\log\left|1+2u+3u^2\right] - \frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3u+1}{\sqrt{2}}\right) + C$$

Question 19:

Obtain an integral (or anti – derivative) of the $\frac{6u+7}{\sqrt{(u-5)(u-4)}}$

Answer 19:

Suppose, $\frac{6u+7}{\sqrt{(u-5)(u-4)}} = \frac{6u+7}{\sqrt{u^2-9u+20}}$

$$Suppose,\ 6u+7 = A\frac{d}{du}(u^2-9u+20)+B$$
$$6u+7 = A(2u-9)+B$$

Equate the coefficients of u and the constants on both the sides, we get,

2 A = 6 => A = 3

– 9 + B = 7 => B = 34

6 u + 7 = 3 (2 u – 9) + 34

$$\int \frac{6u+7}{\sqrt{u^2-9u+20}}\,du = \int \frac{3(2u-9)+34}{\sqrt{u^2-9u+20}}\,du$$
$$= 3\int \frac{2u-9}{\sqrt{u^2-9u+20}}\,du + 34\int \frac{1}{\sqrt{u^2-9u+20}}\,du$$

$$Suppose,\ I_1 = \int \frac{2u-9}{\sqrt{u^2-9u+20}}\,du \ and\ I_2 = \int \frac{1}{\sqrt{u^2-9u+20}}\,du$$
$$\int \frac{6u+7}{\sqrt{u^2-9u+20}}\,du = 3I_1 + 34I_2$$
$$I_1 = \int \frac{2u-9}{\sqrt{u^2-9u+20}}\,du$$
$$Suppose,\ u^2-9u+20 = z(2u-9)\,du = dz$$
$$I_1 = \frac{dz}{\sqrt{z}}$$
$$I_1 = 2\sqrt{z}$$
$$I_1 = 2\sqrt{u^2-9u+20}$$
$$and\ I_2 = \int \frac{1}{\sqrt{u^2-9u+20}}\,du$$
$$u^2-9u+20\ can\ also\ be\ written\ as\ u^2-9u+20+\frac{81}{4}-\frac{81}{4}$$

$$Therefore,\ u^2-9u+20+\frac{81}{4}-\frac{81}{4}$$
$$= \left(u-\frac{9}{2}\right)^2 - \frac{1}{4}$$
$$= \left(u-\frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$
$$I_2 = \int \frac{1}{\sqrt{\left(u-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}\,du$$
$$I_2 = \log\left|\left(u-\frac{9}{2}\right)+\sqrt{u^2-9u+20}\right| + C \ldots..(3)$$

Substituting equations (2) and (3) in equation (1), we get,

$$\int \frac{6u+7}{\sqrt{u^2-9u+20}}\,du = 3\left[2\sqrt{u^2-9u+20}\right] + 34\log\left[\left(\left(u-\frac{9}{2}\right)+\sqrt{u^2-9u+20}\right)\right] + C$$
$$= 6\sqrt{u^2-9u+20} + 34\log\left[\left(\left(u-\frac{9}{2}\right)+\sqrt{u^2-9u+20}\right)\right] + C$$

Question 20:

Obtain an integral (or anti – derivative) of the $\frac{u+2}{\sqrt{4u-u^2}}$

Answer 20:

Suppose, $u+2 = A\frac{d}{du}(4u-u^2)+B$

$$(u+2) = A(4-2u)+B$$

Equate the coefficients of u and the constants on both the sides, we get,

$$-2A = 1 => A = -\frac{1}{2}$$
$$4A+B = 2 => B = 4$$
$$(u+2) = -\frac{1}{2}(4-2u)+4$$
$$\int \frac{u+2}{\sqrt{4u-u^2}}\,du = \int \frac{-\frac{1}{2}\,(4-2u)+4}{\sqrt{4u-u^2}}\,du$$
$$= -\frac{1}{2}\int \frac{4-2u}{\sqrt{4u-u^2}}\,du + 4\int \frac{1}{\sqrt{4u-u^2}}\,du$$

$$Suppose,\ I_1 = \int \frac{4-2u}{\sqrt{4u-u^2}}\,du \ and\ I_2 = \int \frac{1}{\sqrt{4u-u^2}}\,du$$
$$\int \frac{u+2}{\sqrt{4u-u^2}}\,du = -\frac{1}{2}I_1 + 4I_2 \ldots..(1)$$
$$Then,\ I_1 = \int \frac{4-2u}{\sqrt{4u-u^2}}\,du$$
$$Suppose,\ 4u-u^2 = z$$
$$(4-2u)\,du = dz$$
$$I_1 = \frac{dz}{\sqrt{z}} = 2\sqrt{z} = 2\sqrt{4u-u^2} \ldots..(2)$$
$$I_2 = \int \frac{1}{\sqrt{4u-u^2}}\,du$$
$$Suppose,\ 4u-u^2 = -(-4u+u^2)$$
$$(4-2u)\,du = -(-4u+u^2+4-4)$$
$$= 4-(u-2)^2$$
$$I_2 = \int \frac{1}{\sqrt{(2)^2-(u-2)^2}}\,du = \sin^{-1}\left(\frac{u-2}{2}\right) \ldots..(3)$$

Substituting equations (2) and (3) in equation (1), we get,

$$\int \frac{u+2}{\sqrt{4u-u^2}}\,du = -\frac{1}{2}\left(2\sqrt{4u-u^2}\right) + 4\sin^{-1}\left(\frac{u-2}{2}\right) + C$$
$$= -\sqrt{4u-u^2} + 4\sin^{-1}\left(\frac{u-2}{2}\right) + C$$

Question 21:

Question 21:

Obtain an integral (or anti – derivative) of the $\frac{u+2}{\sqrt{4u^2+2u+3}}$

Answer 21:

$$\int \frac{u+2}{\sqrt{u^2+2u+3}}\,du = \frac{1}{2}\int \frac{2(u+2)}{\sqrt{u^2+2u+3}}\,du$$
$$= \frac{1}{2}\int \frac{2u+4}{\sqrt{u^2+2u+3}}\,du$$
$$= \frac{1}{2}\int \frac{2u+2}{\sqrt{u^2+2u+3}}\,du + \frac{1}{2}\int \frac{2}{\sqrt{u^2+2u+3}}\,du$$
$$= \frac{1}{2}\int \frac{2u+2}{\sqrt{u^2+2u+3}}\,du + \int \frac{1}{\sqrt{u^2+2u+3}}\,du$$
$$Suppose,\ I_1 = \int \frac{2u+2}{\sqrt{u^2+2u+3}}\,du\ and\ I_2 = \int \frac{1}{\sqrt{u^2+2u+3}}\,du$$
$$\int \frac{u+2}{\sqrt{u^2+2u+3}}\,du = \frac{1}{2}I_1 + I_2$$

$$I_1 = \int \frac{2u+2}{\sqrt{u^2+2u+3}}\,du$$
$$Suppose,\ u^2+2u+3 = z$$
$$(2u+2)\,du = dz$$
$$I_1 = \int \frac{dz}{\sqrt{z}} = 2\sqrt{z} = 2\sqrt{u^2+2u+3}\ldots..(2)$$
$$I_2 = \int \frac{1}{\sqrt{u^2+2u+3}}\,du$$
$$u^2+2u+3 = u^2+2u+1+2 = (u+1)^2+(\sqrt{2})^2$$
$$I_2 = \int \frac{1}{\sqrt{(u+1)^2+(\sqrt{2})^2}}\,du = log\left|(u+1)+\sqrt{u^2+2u+3}\right|\ldots..(3)$$

Substituting equations (2) and (3) in equation (1), we get,

$$\int \frac{u+2}{\sqrt{u^2+2u+3}}\,du = \frac{1}{2}\left(2\sqrt{u^2+2u+3}\right) + log\ \left|(u+1)+\sqrt{u^2+2u+3}\right| + C$$
$$= \sqrt{u^2+2u+3} + log\left|(u+1)+\sqrt{u^2+2u+3}\right| + C$$

Question 22:

Obtain an integral (or anti – derivative) of the $\frac{u+2}{\sqrt{u^2-2u-5}}$

Answer 22:

$$Suppose,\ (u+3) = A\frac{d}{du}(u^2-2u-5)$$
$$(u+3) = A(2u-2)+B$$

Equate the coefficients of u and the constants on both the sides, we get,

$$2A = 1 => A = \frac{1}{2}$$
$$-2A+B = 3 => B = 4$$
$$(u+3) = \frac{1}{2}(2u-2)+4$$
$$\int \frac{u+3}{\sqrt{u^2-2u-5}}\,du = \int \frac{\frac{1}{2}(2u-2)+4}{u^2-2u-5}\,du$$
$$= \frac{1}{2}\int \frac{(2u-2)+4}{u^2-2u-5}\,du + 4\int \frac{1}{u^2-2u-5}du$$

$$Suppose,\ I_1 = \int \frac{(2u-2)+4}{u^2-2u-5}\,du\ and\ I_2 = \int \frac{1}{u^2-2u-5}du$$
$$\int \frac{u+3}{\sqrt{u^2-2u-5}}\,du = \frac{1}{2}I_1 + 4I_2\ldots.(1)$$
$$Then, I_1 = \int \frac{(2u-2)+4}{u^2-2u-5}\,du$$
$$Suppose,\ u^2-2u-5 = Z$$
$$(2u-2)\,du = dz$$
$$I_1 = \int \frac{dz}{z}$$
$$= log\,|z| + C$$
$$= log\left|u^2-2u-5\right| + C\ldots.(2)$$

$$I_2 = \int \frac{1}{u^2-2u-5}du$$
$$= \int \frac{1}{(u^2-2u+1)-6}du$$
$$= \int \frac{1}{(u-1)^2+(\sqrt{6})^2}\,du$$
$$= \frac{1}{2\sqrt{6}}log\left(\frac{u-1-\sqrt{6}}{u-1+\sqrt{6}}\right)\ldots...(3)$$

Using equations (2) and (3) in equation (1), we get,

$$\int \frac{u+3}{\sqrt{u^2-2u-5}}\,du = \frac{1}{2}log\left|u^2-2u-5\right| + 4\left[\frac{1}{2\sqrt{6}}log\left(\frac{u-1-\sqrt{6}}{u-1+\sqrt{6}}\right)\right] + C$$
$$= \frac{1}{2}log\left|u^2-2u-5\right| + \frac{2}{\sqrt{6}}log\left(\frac{u-1-\sqrt{6}}{u-1+\sqrt{6}}\right)$$

Question 23:

Obtain an integral (or anti – derivative) of the $\frac{5u+3}{\sqrt{u^2+4u+10}}$

Answer 23:

$$Suppose,\ 5u+3 = A\frac{d}{du}(u^2+4u+10)+B$$
$$5u+3 = A(2u+4)+B$$

Equate the coefficients of u and the constants on both the sides, we get,

$$2A = 5 => A = \frac{5}{2}$$
$$4A+B = 3 => B =-7$$
$$5u+3 = \frac{5}{2}(2u+4)-7$$
$$\int \frac{5u+3}{\sqrt{u^2+4u+10}}\,du = \frac{\frac{5}{2}((2u+4)-7)}{\sqrt{u^2+4u+10}}\,du$$
$$= \frac{5}{2}\int \frac{2u+4}{\sqrt{u^2+4u+10}}\,du-7\int \frac{1}{\sqrt{u^2+4u+10}}\,du$$
$$Suppose,\ I_1 = \int \frac{2u+4}{\sqrt{u^2+4u+10}}\,du\ and\ I_2 = \int \frac{1}{\sqrt{u^2+4u+10}}\,du$$
$$\int \frac{5u+3}{\sqrt{u^2+4u+10}}\,du = \frac{5}{2}I_1-7I_2\quad\ldots..(1)$$

$$I_1 = \int \frac{2u+4}{\sqrt{u^2+4u+10}}\,du$$
$$Suppose,\ u^2+4u+10 = z$$
$$(2u+4)\,du = dz$$
$$I_1 = \int \frac{dz}{\sqrt{z}} = 2\sqrt{z} = 2\sqrt{u^2+4u+10}\ldots..(2) I_2 = \int \frac{1}{\sqrt{u^2+4u+10}}\,du$$
$$= \int \frac{1}{\sqrt{(u^2+4u+4)+6}}\,du$$
$$= \int \frac{1}{(u+2)^2+(\sqrt{6})^2}\,du$$
$$= log\left|(u+2)\sqrt{u^2+4u+10}\right|\ldots..(3)$$

Substituting equations (2) and (3) in equation (1), we get,

$$\int \frac{5u+3}{\sqrt{u^2+4u+10}}\,du = \frac{5}{2}\left[2\sqrt{u^2+4u+10}\right]-7log\left|(u+2)+\sqrt{u^2+4u+10}\right| + C$$
$$= 5\ \sqrt{u^2+4u+10}-7log\left|(u+2)+\sqrt{u^2+4u+10}\right| + C$$

Question 24: Which of the following below is the answer for $\int \frac{du}{u^2+2u+2}\, du$

$(a) u tan^{-1}(u+1)+C$
$(b) tan^{-1}(u+1)+C$
$(c) (u+1)tan^{-1}(u)+C$
$(d) tan^{-1}(u)+C$

Answer 24:

$$\int \frac{du}{u^2+2u+2}\, du = \int \frac{du}{(u^2+2u+1)+1}$$
$$= \int \frac{1}{(u+1)^2+(1)^2}\, du$$
$$= \left[tan^{-1}(u+1)\right]+C$$

Thus, (b) is the correct answer.

Question 25: Which of the following below is the answer for $\int \frac{du}{\sqrt{9u-4u^2}}\, du$

$(a) \frac{1}{9} sin^{-1}\frac{9u-8}{8}+C$
$(b) \frac{1}{2} sin^{-1}\frac{8u-9}{9}+C$
$(c) \frac{1}{3} sin^{-1}\frac{9u-8}{8}+C$
$(d) \frac{1}{2} sin^{-1}\frac{9u-8}{8}+C$

Answer 25:

$$\int \frac{du}{\sqrt{9u-4u^2}}\, du$$
$$= \int \frac{du}{\sqrt{-4\left(u^2-\frac{9}{4}u\right)}}$$
$$= \int \frac{du}{\sqrt{-4\left(u^2-\frac{9}{4}u+\frac{81}{64}-\frac{81}{64}\right)}}$$
$$= \int \frac{1}{\sqrt{-4}\left[\left(u-\frac{9}{8}\right)^2-\left(\frac{9}{8}\right)^2\right]}\, du$$
$$= \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{9}{8}\right)^2-\left(u-\frac{9}{8}\right)^2}}\, du$$
$$= \frac{1}{2}\left[sin^{-1}\left(\frac{u-\frac{9}{8}}{\frac{9}{8}}\right)\right]+C$$
$$= \frac{1}{2} sin^{-1}\left(\frac{8u-9}{9}\right)+C$$

Thus, (b) is the correct answer.

Exercise 7.4

Question 1:

Obtain an integral (or anti – derivative) of the following rational number $\frac{u}{(u+1)(u+2)}$

Answer 1:

Suppose, $\frac{u}{(u+1)(u+2)} = \frac{A}{u+1}+\frac{B}{u+2}$

$$=> u = A(u+2)+B(u+1)$$

Equate the coefficients of u and the constants on both the sides, we get,

A + B = 1

2 A + B = 0

On solving, we get,

A = – 1 and B = 2

$$\frac{u}{(u+1)(u+2)} = \frac{-1}{u+1}+\frac{2}{u+2}$$
$$=> \int \frac{u}{(u+1)(u+2)}\, du = \frac{-1}{u+1}+\frac{2}{u+2}\, du$$
$$= -log\left|u+1\right|+2log\left|u+2\right|+C$$
$$= log(u+2)^2 - log\left|u+1\right|+C$$
$$= log\frac{(u+1)^2}{(u+1)}+C$$

Question 2:

Obtain an integral (or anti – derivative) of the following rational number $\frac{1}{u^2-9}$

Answer 2:

Suppose, $\frac{1}{(u+3)(u-3)} = \frac{A}{u+3} + \frac{B}{u-3}$

$$1 = A(u-3) + B(u+3)$$

Equate the coefficients of u and the constants on both the sides, we get,

A + B = 0

– 3 A + 3 B = 1

On solving, we get

$$A = -\frac{1}{6} \text{ and } B = \frac{1}{6}$$
$$\frac{1}{(u+3)(u-3)} = \frac{-1}{6(u+3)} + \frac{1}{6(u-3)}$$
$$\Rightarrow \int \frac{1}{u^2-9}\, du = \int \left(\frac{-1}{6(u+3)} + \frac{1}{6(u-3)} \right) du$$
$$= -\frac{1}{6} \log|u+3| + \frac{1}{6} \log|u-3| + C$$
$$= \frac{1}{6} \log\left|\frac{(u-3)}{(u+3)}\right| + C$$

Question 3:

Obtain an integral (or anti – derivative) of the following rational number $\frac{3u-1}{(u-1)(u-2)(u-3)}$

Answer 3:

Suppose, $\frac{3u-1}{(u-1)(u-2)(u-3)} = \frac{A}{(u-1)} + \frac{B}{(u-2} + \frac{C}{(u-3)}$

$$3u-1 = A(u-2)(u-3) + B(u-1)(u-3) + C(u-1)(u-2) \ldots\ldots (1)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B + C = 0

– 5 A – 4 B – 3 C = 1

6 A + 3 B + 2 C = – 1

On solving, we get,

A = 1, B = – 5, and C = 4

$$\frac{3u-1}{(u-1)(u-2)(u-3)} = \frac{1}{(u-1)} - \frac{5}{(u-2} + \frac{4}{(u-3)}$$
$$\int \frac{3u-1}{(u-1)(u-2)(u-3)}\, du = \int \left\{ \frac{1}{(u-1)} - \frac{5}{(u-2} + \frac{4}{(u-3)} \right\} du$$
$$= \log|u-1| - 5\log|u-2| + 4\log|u-3| + C$$

Question 4:

Obtain an integral (or anti – derivative) of the following rational number $\frac{u}{(u-1)(u-2)(u-3)}$

Answer 4:

Suppose, $\frac{u}{(u-1)(u-2)(u-3)} = \frac{A}{(u-1)} + \frac{B}{(u-2} + \frac{C}{(u-3)}$

$$u = A(u-2)(u-3) + B(u-1)(u-3) + C(u-1)(u-2) \ldots\ldots (1)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B + C = 0

– 5 A – 4 B – 3 C = 1

6 A + 3 B + 2 C = 0

On solving, we get,

$$A = \frac{1}{2},\ B = -2 \text{ and } C = \frac{3}{2}$$
$$\frac{u}{(u-1)(u-2)(u-3)} = \frac{1}{2(u-1)} - \frac{2}{(u-2} + \frac{3}{2(u-3)}$$
$$\int \frac{u}{(u-1)(u-2)(u-3)}\, du = \int \left\{ \frac{1}{2\,(u-1)} - \frac{2}{(u-2)} + \frac{3}{2(u-3)} \right\} du$$
$$= \frac{1}{2}\log|u-1| - 2\log|u-2| + \frac{3}{2}\log|u-3| + C$$

Question 5:

Obtain an integral (or anti – derivative) of the following rational number $\frac{2u}{u^2+3u+2}$

Answer 5:

Answer 5:

Suppose, $\frac{2u}{u^2+3u+2} = \frac{A}{(u+1)} + \frac{B}{(u+2)}$

$$2u = A(u+2) + B(u+1) \ldots . (1)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B = 2

2 A + B = 0

On solving, we get,

A = – 2 and B = 4

$$\frac{2u}{(u+1)(u+2)} = \frac{-2}{(u+1)} + \frac{4}{(u+1)}$$
$$\int \frac{2u}{(u+1)(u+2)}\, du = \int \left\{ \frac{4}{(u+1)} - \frac{2}{(u+1)} \right\} du$$
$$= 4log\,|u+2| - 2log\,|u+1| + C$$

Question 6:

Obtain an integral (or anti – derivative) of the following rational number $\frac{1-u^2}{u(1-2u)}$

Answer 6:

$\frac{1-u^2}{u(1-2u)}$ is not a proper fraction.

Dividing (1 – u ^{2}) by u (1 – 2 u), we get,

$$\frac{1-u^2}{u(1-2u)} = \frac{1}{2} + \frac{1}{2}\left(\frac{2-u}{u(1-2u)}\right)$$
$$Suppose,\ \frac{2-u}{u(1-2u)} = \frac{A}{u} + \frac{B}{(1-2u)}$$
$$(2-u) = A(1-2u) + Bu \ldots \ldots . (1)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

– 2 A + B = – 1

And A = 2

On solving, we get,

A = 2 and B = 3

$$\frac{2-u}{u(1-2u)} = \frac{2}{u} + \frac{3}{(1-2u)}$$
$$Using\ in\ equation\ (1), we\ get,$$
$$\frac{1-u^2}{u(1-2u)} = \frac{1}{2} + \frac{1}{2}\left\{\frac{2}{u} + \frac{3}{(1-2u)}\right\}$$
$$= \frac{u}{2} + log\,|u| + \frac{3}{2(-2)}\, log\,|1-2u| + C$$
$$= \frac{u}{2} + log\,|u| - \frac{3}{4}\, log\,|1-2u| + C$$

Question 7:

Obtain an integral (or anti – derivative) of the following rational number $\frac{u}{(u^2+1)(u-1)}$

Answer 7:

Suppose, $\frac{u}{(u^2+1)(u-1)} = \frac{Au+B}{(u^2+1)} + \frac{C}{u-1}$

$$u = (Au+B)(u-1) + C(u^2+1)$$
$$u = Au^2 - Au + Bu - B + Cu^2 + C$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + C = 0

– A + B = 1

– B + C = 0

On solving, we get,

$$A = -\frac{1}{2},\ B = \frac{1}{2}, and\ C = \frac{1}{2}$$
$$Using\ equation\ (1),\ we\ get$$
$$\frac{u}{(u^2+1)(u-1)} = \frac{\left(-\frac{1}{2}u + \frac{1}{2}\right)}{(u^2+1)} + \frac{\frac{1}{2}}{(u-1)}$$

$$\int \frac{u}{(u^2+1)(u-1)}\, du = -\frac{1}{2}\int \frac{u}{(u^2+1)}\, du + \frac{1}{2}\int \frac{1}{(u^2+1)} + \frac{1}{2}\int \frac{1}{2}\int \frac{1}{u-1}\, du$$
$$= -\frac{1}{4}\int \frac{2u}{(u^2+1)} + \frac{1}{2}tan^{-1}u + \frac{1}{2}log\,|u-1| + C$$
$$\int \frac{2u}{(u^2+1)}\, du, let\ (u^2+1) = z => 2u\, du = dz$$
$$\int \frac{2u}{(u^2+1)}\, du = \int \frac{dz}{z} = log\,|z| = log\ |u^2+1|$$
$$\int \frac{u}{(u^2+1)(u-1)}\, du = -\frac{1}{4}log\,|(u^2+1)| + \frac{1}{2}tan^{-1}u + \frac{1}{2}log\,|u-1| + C$$
$$= \frac{1}{2}log\,|u-1| - \frac{1}{4}log\,|(u^2+1)| + \frac{1}{2}tan^{-1}u + C$$

Question 8:

Obtain an integral (or anti – derivative) of the following rational number $\frac{u}{(u-1)^2(u+2)}$

Answer 8:

$$\frac{u}{(u-1)c^2(u+2)} = \frac{A}{(u-1)} + \frac{B}{(u-1)^2} + \frac{C}{u+2}$$
$$u = A(u-1)(u+2) + B(u+2) + C(u-1)^2$$

Putting u = 1, we get,

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + C = 0

A + B – 2 C = 1

– 2 A + 2 B + C = 0

On solving, we get,

$$A = \frac{2}{9},\ B = \frac{1}{3}\ and\ C = -\frac{2}{9}\ \frac{u}{(u-1)^2(u+2)} = \frac{2}{9\ (u-1)} + \frac{1}{3\ (u-1)^2} - \frac{2}{9(u+2)}$$
$$\int \frac{u}{(u-1)^2(u+2)}\ du = \frac{2}{9}\int \frac{1}{(u-1)}\ du + \frac{1}{3}\int \frac{1}{(u-1)^2}\ du - \frac{2}{9}\int \frac{1}{(u+2)}\ du$$
$$= \frac{2}{9}log\ |u-1| + \frac{1}{3}\left(\frac{-1}{u-1}\right) - \frac{2}{9}log\,|u+2| + C$$
$$= \frac{2}{9}log\ \left|\frac{u-1}{u+2}\right| - \frac{1}{3(u-1)} + C$$

Question 9:

Obtain an integral (or anti – derivative) of the following rational number $\frac{3u+5}{u^3-u^2-u+1}$

Answer 9:

$$\frac{3u+5}{u^3-u^2-u+1} = \frac{3u+5}{(u-1)^2(u+1)}$$
$$Suppose,\ \frac{3u+5}{(u-1)^2(u+1)} = \frac{A}{(u-1)} + \frac{B}{(u-1)^2} + \frac{C}{(u+1)}$$
$$3u + 5 = A(u-1)(u+1) + B(u+1) + C(u-1)^2$$
$$3u + 5 = A(u^2-1) + B(u+1) + C(u^2 + 1-2u)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + C = 0

B – 2 C = 3

– A + B + C = 5

On solving, we get,

B = 4

$$A = -\frac{1}{2}\ and\ C = \frac{1}{2}$$
$$\frac{3u+5}{(u-1)^2(u+1)} = \frac{-1}{2(u-1)} + \frac{4}{(u-1)^2} + \frac{1}{2(u+1)}$$
$$\int \frac{3u+5}{(u-1)^2(u+1)}\ du = -\frac{1}{2}\int \frac{1}{(u-1)}\ du + 4\int \frac{1}{(u-1)^2}\ du + \frac{1}{2}\int \frac{1}{(u+1)}\ du$$
$$= -\frac{1}{2}log\ |u-1| + 4\left(\frac{-1}{u-1}\right) + \frac{1}{2}log\,|u+1| + C$$
$$= \frac{1}{2}log\left|\frac{u+1}{u-1}\right| - \frac{4}{(u-1)} + C$$

Question 10:

Obtain an integral (or anti – derivative) of the following rational number $\frac{2u-3}{(u^2-1)(2u+3)}$

Answer 10:

$$\frac{2u-3}{(u^2-1)(2u+3)} = \frac{2u-3}{(u+1)(u-1)(2u+3)}$$

Suppose, $\frac{2u-3}{(u^2-1)(2u+3)} = \frac{A}{(u+1)} + \frac{B}{(u-1)} + \frac{C}{(2u+3)}$

$$(2u-3) = A\ (u-1)(2u+3) + B\ (u+1)(2u+3) + C\ (u+1)(u-1)$$
$$(2u-3) = A\ (2u^2 + u-3) + B\ (2u^2 + 5u + 3) + C\ (u^2-1)$$
$$(2u-3) = (2A + 2B + C)u^2 + (A + 5B)u + (-3A + 3B-C)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

2A + 2B + C = 1

A + 5B = 2

– 3A + 3B – C = – 3

On solving, we get,

$$\frac{2u-3}{(u+1)(u-1)(2u+3)} = \frac{5}{2(u+1)} - \frac{1}{10(u-1)} - \frac{24}{5(2u+3)}$$
$$\frac{2u-3}{(u+1)(u-1)(2u+3)} = \frac{5}{2}\int\frac{1}{(u+1)}\,du - \frac{1}{10}\int\frac{1}{(u-1)}\,du - \frac{24}{5}\int\frac{1}{(2u+3)}\,du$$
$$= \frac{5}{2}log\ |u+1| - \frac{1}{10}log\ |u-1| - \frac{24}{5\times2}log\ |2u+3| + C$$
$$= \frac{5}{2}log\ |u+1| - \frac{1}{10}log\ |u-1| - \frac{12}{5}log\ |2u+3| + C$$

Question 11:

Obtain an integral (or anti – derivative) of the following rational number $\frac{5u}{(u+1)(u^2-4)}$

Answer 11:

$$\frac{5u}{(u+1)(u^2-4)} = \frac{5u}{(u+1)(u+2)(u-2)}$$
$$Suppose,\ \frac{5u}{(u+1)(u+2)(u-2)} = \frac{A}{(u+1)} + \frac{B}{(u+2)} + \frac{C}{(u-2)}$$
$$5u = A(u+2)(u-2) + B(u+1)(u-2) + C(u+1)(u+2)\ldots.(1)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B + C = 0

– B + 3C = 5 and

– 4A – 2B + 2C = 0

On solving, we get,

$$A = \frac{5}{3}, B = -\frac{5}{2}\ and\ C = \frac{5}{6}$$
$$\frac{5u}{(u+1)(u+2)(u-2)} = \frac{5}{3(u+1)} - \frac{5}{2(u+2)} + \frac{5}{6(u-2)}$$
$$\int\frac{5u}{(u+1)(u+2)(u-2)}\,du = \frac{5}{3}\frac{1}{(u+1)}\,du - \frac{5}{2}\frac{1}{(u+2)}\,du + \frac{5}{6}\frac{1}{(u-2)}\,du$$
$$= \frac{5}{3}log\ |u+1| - \frac{5}{2}log\ |u+2| + \frac{5}{6}log\ |u-2| + C$$

Question 12:

Obtain an integral (or anti – derivative) of the following rational number $\frac{u^3+u+1}{u^2-1}$

Answer 12:

$\frac{u^3+u+1}{u^2-1}$ is not a proper fraction.

So, dividing (u^3 + u + 1) by u^2 – 1, we get,

$$\frac{u^2+u+1}{u^2-1} = u + \frac{2u+1}{u^2-2}$$
$$Suppose,\ \frac{2u+1}{u^2-2} = \frac{A}{(u+1)} + \frac{B}{(u-1)}$$
$$2u+1 = A(u-1) + B(u+1)\ldots..(1)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B = 2

– A + B = 1

On solving, we get,

$$A = \frac{1}{2}\ and\ B = \frac{3}{2}$$
$$\frac{u^2+u+1}{u^2-1} = u + \frac{1}{2(u+1)} + \frac{3}{2(u-1)}$$
$$Integrating\ on\ both\ the\ sides,\ we\ get, \int\frac{u^2+u+1}{u^2-1}\,du = \int u\,du + \frac{1}{2}\int\frac{1}{u+1}\,du + \frac{3}{2}\frac{1}{(u-1)}\,du$$
$$= \frac{u^2}{2} + \frac{1}{2}log\ |u+1| - \frac{3}{2}log\ |u-1| + C$$

Question 13:

Obtain an integral (or anti – derivative) of the following rational number $\frac{2}{(1-u)(1+u^2)}$

Answer 13:

Suppose, $\frac{2}{(1-u)(1+u^2)} = \frac{A}{1-u} + \frac{Bu+C}{1+u^2}$

$$2 = A(1+u^2) + (Bu+C)(1-u)$$
$$2 = A + Au^2 + Bu - Bu^2 + C - Cu$$

$2 = A + Au^2 + Bu - Bu^2 + C - Cu$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A – B = 0

B – C = 0

A + C = 2

On solving, we get,

A = 1, B = 1 and C = 1

$$\frac{2}{(1-u)(1+u^2)} = \frac{1}{1-u} + \frac{u+1}{1+u^2}$$
$$\int \frac{2}{(1-u)(1+u^2)}\, du = \int \frac{1}{1-u}\, du + \int\int \frac{u}{1+u^2}\, du + \int \frac{1}{1+u^2}\, du$$
$$= -log\ |u-1| + \frac{1}{2} log\ \left|1+u^2\right| + tan^{-1} u + C$$

Question 14:

Obtain an integral (or anti – derivative) of the following rational number $\frac{3u-1}{(u+2)^2}$

Answer 14:

Suppose, $\frac{3u-1}{(u+2)^2} = \frac{A}{(u+2)} + \frac{B}{(u+2)^2}$
$$3u-1 = A(u+2) + B$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A = 3

2A + B = – 1

B = – 7

$$\frac{3u-1}{(u+2)^2} = \frac{3}{(u+2)} - \frac{7}{(u+2)^2}$$
$$3u-1 = A(u+2) + B\frac{3u-1}{(u+2)^2} = 3\int \frac{1}{(u+2)}\, du - 7\int \frac{u}{(u+2)^2}\, du$$
$$= 3\ log\ |u+2| - 7\left(\frac{-1}{(u+2)}\right) + C$$
$$= 3\ log\ |u+2| + \left(\frac{7}{(u+2)}\right) + C$$

Question 15:

Obtain an integral (or anti – derivative) of the following rational number $\frac{1}{u^4-1}$

Answer 15:

$$\frac{1}{u^4-1} = \frac{1}{(u^2-1)(u^4+1)} = \frac{1}{(u+1)(u-1)(1+u^2)}$$
$$Suppose,\ \frac{1}{(u+1)(u-1)(1+u^2)} = \frac{A}{(u+1)} + \frac{B}{(u-1)} + \frac{Cu+D}{(u^2+1)}$$
$$1 = A(u+1)(u^2+1) + B(u+1)(u^2+1) + (Cu+D)(u^2-1)$$
$$1 = A(u^3+u-u^2-1) + B(u^3+u+u^2+1) + Cu^2 + Du^2 - Cu - D$$
$$1 = (A+B+C)u^3 + (-A+B+D)u^2 + (A+B-C)u + (-A+B-D)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B + C = 0

– A + B + D = 0

A + B – C = 0

– A + B – D = 1

On solving, we get,

$$A = -\frac{1}{4}, B = \frac{1}{4}, C = 0\ and\ D = -\frac{1}{2}$$
$$\frac{1}{u^4-1} = \frac{-1}{4(u+1)} + \frac{1}{4(u-1)} - \frac{1}{2(u^2+1)}$$
$$\int \frac{1}{u^4-1}\, du = -\frac{1}{4}\int \frac{1}{(u+1)}\, du + \frac{1}{4}\int \frac{1}{(u-1)}\, du + \frac{1}{2}\int \frac{1}{(u^2+1)}$$
$$= -\frac{1}{4} log\ |u+1| + \frac{1}{4} log\ |u-1| - \frac{1}{2} tan^{-1} u + C$$
$$= \frac{1}{4} log\ \left|\frac{u-1}{u+1}\right| - \frac{1}{2} tan^{-1} u + C$$

Question 16:

Obtain an integral (or anti – derivative) of the following rational number $\frac{1}{\ldots}$

$u(u^m+1)$

[Hint: multiply denominator and numerator by u $^{n-1}$ and put $u^n = z$]

Answer 16:

$\frac{1}{u(u^m+1)}$

Multiplying denominator and numerator by u $^{n-1}$, we get,

$$\frac{1}{u(u^m+1)} = \frac{u^{m-1}}{u^{m-1}.u(u^m+1)} = \frac{u^{m-1}}{u^m(u^m+1)}$$
$$Suppose,\ u^m = z => u^{m-1}\ du = dz$$
$$\int \frac{1}{u(u^m+1)}\ du = \int \frac{u^{m-1}}{u^m(u^m+1)}\ du = \frac{1}{m}\int \frac{1}{z(z+z)}\ du$$
$$Suppose,\ \frac{1}{z(z+z)} = \frac{A}{z} + \frac{B}{(z+1)}$$
$$1 = A(1+z) + Bz$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A = 1 and B = – 1

$$\frac{1}{z(z+z)} = \frac{1}{z} - \frac{1}{(z+1)}$$
$$\int \frac{1}{u(u^m+1)}\ du = \frac{1}{m}\int \left\{\frac{1}{z} - \frac{1}{(z+1)}\right\} + C$$
$$= \frac{1}{m}[log\ |u^m| - log\ |u^m+1|] + C$$
$$= \frac{1}{m} log\ \left|\frac{u^m}{u^m+1}\right|$$

Question 17:

Obtain an integral (or anti – derivative) of the following rational number $\frac{cos\ u}{(1-sin\ u)(2-sin\ u)}$

[Hint: Put sin u = z]

Answer 17:

$\frac{cos\ u}{(1-sin\ u)(2-sin\ u)}$

Suppose, sin u = z => cos u du = dz

$$\int \frac{cos\ u}{(1-sin\ u)(2-sin\ u)}\ du = \int \frac{dz}{(1-z)(2-z)}$$
$$Suppose,\ \frac{1}{(1-z)(2-z)} = \frac{A}{(1-z)} + \frac{B}{(2-z)}$$
$$1 = A(2-z) + B(1-z)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

– 2A – B = 0

2A + B = 1

On solving, we get,

A = 1 and B = – 1

$$\frac{1}{(1-z)(2-z)} = \frac{1}{(1-z)} - \frac{1}{(2-z)}$$
$$\int \frac{cos\ u}{(1-sin\ u)(2-sin\ u)}\ du = \int \left\{\frac{1}{(1-z)} - \frac{1}{(2-z)}\right\}\ dz$$
$$= -log\ |1-z| + log\ |2-z| + C$$
$$= log\ \left|\frac{2-z}{1-z}\right| + C$$
$$= log\ \left|\frac{2-sin\ u}{1-sin\ u}\right| + C$$

Question 18:

Obtain an integral (or anti – derivative) of the following rational number $\frac{(u^2+1)(u^2+2)}{(u^2+3)(u^2+4)}$

Answer 18:

$$\frac{(u^2+1)(u^2+2)}{(u^2+3)(u^2+4)} = 1 - \frac{(4u^2+10)}{(u^2+3)(u^2+4)}$$
$$Suppose,\ \frac{(4u^2+10)}{(u^2+3)(u^2+4)} = \frac{Au+B}{(u^2+3)} + \frac{Cu+D}{(u^2+4)}$$
$$(4u^2+10) = (Au+B)(u^2+4) + (Cu+D)(u^2+3)$$
$$(4u^2+10) = Au^3 + 4Au + Bu^2 + 4B + Cu^3 + 3Cu + Du^2 + 3D$$
$$(4u^2+10) = (A+C)u^3 + (B+D)u^2 + (4A+3C)u + (4B+3D)$$

Equate the coefficients of u^3, u^2, u and the constants on both the sides, we get,

A + C = 0

B + D = 4

B + D = 4

4 A + 3 C = 0

4 B + 3 D = 10

On solving, we get,

A = 0, B = – 2, C = 0 and D = 6

$$\frac{(4u^2+10)}{(u^2+3)(u^2+4)} = \frac{-2}{(u^2+3)} + \frac{6}{(u^2+4)}$$
$$\frac{(u^2+1)(u^2+2)}{(u^2+3)(u^2+4)} = 1 - \left(\frac{-2}{(u^2+3)} + \frac{6}{(u^2+4)}\right)$$
$$\int \frac{(u^2+1)(u^2+2)}{(u^2+3)(u^2+4)}\, du = \int \left\{1 + \frac{2}{(u^2+3)} + \frac{6}{(u^2+4)}\right\}$$
$$= \int \left\{1 + \frac{2}{u^2+(\sqrt{3})^2} - \frac{6}{u^2+2^2}\right\} du$$
$$= u + 2\left(\frac{1}{\sqrt{3}}\, tan^{-1}\frac{u}{\sqrt{3}}\right) - 6\left(\frac{1}{2}\, tan^{-1}\frac{u}{2}\right) + C$$
$$= u + \frac{2}{\sqrt{3}}\, tan^{-1}\frac{u}{\sqrt{3}} - 3\, tan^{-1}\frac{u}{2} + C$$

Question 19:

Obtain an integral (or anti – derivative) of the following rational number $\frac{2\,u}{(u^2+1)(u^2+3)}$

Answer 19:

$\frac{2\,u}{(u^2+1)(u^2+3)}$

Suppose, $u^2 = z$

2u du = dz

$$\int \frac{2\,u}{(u^2+1)(u^2+3)}\, du = \int \frac{dz}{(z+1)(z+3)} \ldots.(1)$$
$$Suppose,\ \frac{1}{(z+1)(z+3)} = \frac{A}{(z+1)} + \frac{B}{(z+3)}$$
$$1 = A\,(z+3) + B\,(z+1) \ldots.(1)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B = 0

3 A + B = 1

On solving, we get,

$$A = \frac{1}{2}\ and\ B = -\frac{1}{2}$$
$$\frac{1}{(z+1)(z+3)} = \frac{1}{2\,(z+1)} - \frac{1}{2\,(z+3)}$$
$$\int \frac{2\,u}{(u^2+1)(u^2+3)}\, du = \int \left\{\frac{1}{2\,(z+1)} - \frac{1}{2\,(z+3)}\right\} dz$$
$$= \frac{1}{2} log\,|(z+1)| - \frac{1}{2} log\,|(z+3)| + C$$
$$= \frac{1}{2} log\left|\frac{z+1}{z+3}\right| + C$$
$$= \frac{1}{2} log\left|\frac{u^2+1}{u^2+3}\right| + C$$

Question 20:

Obtain an integral (or anti – derivative) of the following rational number $\frac{1}{u(u^4-1)}$

Answer 20:

$\frac{1}{u(u^4-1)}$

Multiplying denominator and numerator by u^3, we get,

Misplaced &

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A = – 1 and B = 1

$$\frac{1}{z(z-1)} = \frac{-1}{z} + \frac{1}{z-1}$$
$$= \int \frac{1}{u(u^4-1)}\, du = \frac{1}{4} \int \left(\frac{-1}{z} + \frac{1}{z-1}\right) dz$$
$$= \frac{1}{4}[-log\ |z| + log\ |z-1|] + C$$
$$= \frac{1}{4} log\ \left|\frac{z-1}{z}\right| + C$$

$= \frac{1}{4} log \left|\frac{u^4-1}{u^4}\right| + C$

Question 21:

Obtain an integral (or anti – derivative) of the following rational number $\frac{1}{(e^u-1)}$

Answer 21:

$\frac{1}{(e^u-1)}$

Suppose, $e^u = z$

$e^u\, du = dz$

$$\int \frac{1}{(e^u-1)}\, du = \int \frac{1}{z-1} \times \frac{dz}{z} = \int \frac{1}{z(z-1)}\, dz$$
$$Suppose,\ \frac{1}{z(z-1)} = \frac{A}{z} + \frac{B}{z-1}$$
$$1 = A(z-1) + Bz$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A = – 1 and B = 1

$$\frac{1}{z(z-1)} = \frac{-1}{z} + \frac{1}{z-1}$$
$$\int \frac{1}{z(z-1)}\, du = log \left|\frac{z-1}{z}\right| + C$$
$$= log \left|\frac{e^u-1}{e^u}\right| + C$$

Question 22: Which of the following below is an integral of $\frac{u\ du}{(u-1)(u-2)}$

$$(a) log \left|\frac{(u-1)^2}{u-2}\right| + C$$
$$(b) log \left|\frac{(u-2)^2}{u-2}\right| + C$$
$$(c) log \left|\left(\frac{(u-1)}{u-2}\right)^2\right| + C$$
$$(d) log\ |(u-1)(u-2)| + C$$

Answer 22:

$$Suppose,\ \frac{u\ du}{(u-1)(u-2)} = \frac{A}{(u-1)} + \frac{B}{u-2}$$
$$u = A(u-2) + B(u-1) \ldots . (1)$$

Equate the coefficients of u and the constants on both the sides, we get,

A = – 1 and B = 2

$$\frac{u\ du}{(u-1)(u-2)} = \frac{-1}{(u-1)} + \frac{2}{u-2}$$
$$\int \frac{u\ du}{(u-1)(u-2)}\, d = \left\{\frac{-1}{(u-1)} + \frac{2}{u-2}\right\}\, du$$
$$= -log\ |u-1| + 2\ log\ |u-2| + C$$
$$= log \left|\frac{(u-2)^2}{u-1}\right| + C$$

Hence, option (b) is the correct answer.

Question 23: Which of the following below is an integral of $\int \frac{du}{u(u^2+1)}\, du$

$$(a) log\ |u| - \frac{1}{2} log\ (u^2+1) + C$$
$$(b) log\ |u| + \frac{1}{2} log\ (u^2+1) + C$$
$$(c) - log\ |u| + \frac{1}{2} log\ (u^2+1) + C$$
$$(d) log\ |u| + \frac{1}{2} log\ (u^2+1) + C$$

Answer 23:

$$Suppose,\ \frac{1}{u(u^2+1)} = \frac{A}{u} + \frac{Bu+C}{u^2+1}$$
$$1 = A(u^2+1) + (Bu+C)u$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B = 0

C = 0

A = 1

On solving, we get,

A =1, B = – 1, and C = 0

$\frac{1}{u(u^2+1)} = \frac{1}{u} + \frac{-U}{u^2+1}$

$\int \frac{1}{u(u^2+1)}\ du = \int \left\{ \frac{1}{u} - \frac{u}{u^2+1} \right\}\ du$

$= log\ |u| - \frac{1}{2} log\ |u^2+1| + C$

Hence, option (a) is the correct answer.

Exercise 7.6

Question 1:

Obtain an integral of u sin u.

Answer 1:

Suppose, I = $\int u\ sin\ u\ du$

Integrating the equation by parts by taking u as first function and sin u as second function, we get,

$I = u\ \int sin\ u\ du - \int \left\{ \left(\frac{d}{du}\ u \right) \int sin\ u\ du \right\}\ du$

$= u(-cos\ u) - \int 1.(-cos\ u)\ du = -u\ cos\ u + sin\ u + C$

Question 2:

Obtain an integral of u sin 3u.

Answer 2:

Suppose, I = $\int u\ sin\ u\ du$

Integrating the equation by parts by taking u as first function and sin 3u as second function, we get,

$I = u\ \int sin\ 3u\ du - \int \left\{ \left(\frac{d}{du}\ u \right) \int sin\ 3u\ du \right\}\ du$

$= u(\frac{-cos\ 3u}{3}) - \int 1.(\frac{-cos\ 3u}{3})\ du$

$= \frac{-u\ cos\ 3u}{3} + \frac{1}{9} sin\ 3u + C$

Question 3:

Obtain an integral of $u^2.\ e^u$

Answer 3:

Suppose, I = $\int u^2.\ e^u\ du$

Integrating the equation by parts by taking u^2 as first function and e^u as second function, we get,

$I = u^2 \int e^u\ du - \int \left\{ \left(\frac{d}{du} u^2 \right) \int e^u\ du \right\}\ du$

$= u^2 e^u - \int 2u\ e^u\ du$

$= u^2 e^u - 2 \int u\ e^u\ du$

$Integrating\ by\ parts,\ we\ get$

Integrating by parts, we get,

$$= u^2 e^u - 2\left[u \int e^u\, du - \int\left\{\left(\frac{d}{du}u\right).\int e^u\, du\right\}\right]\, du$$
$$= u^2 e^u - 2\left[u\, e^u - \int e^u\, du\right]$$
$$= u^2 e^u - 2\left[u\, e^u - e^u\right]$$
$$= u^2 e^u - 2u\, e^u - 2e^u + C$$
$$= e^u(u^2 - 2u + 2) + C$$

Question 4:

Obtain an integral of u log u.

Answer 4:

Suppose, I = $\int u\, log\, u\, du$

Integrating the equation by parts by taking log u as first function and u as second function, we get,

$$I = log\, u \int u\, du - \int\left\{\left(\frac{d}{du}log\, u\right)\int u\, du\right\}\, du$$
$$= log\, u.\,\frac{u^2}{2} - \int \frac{1}{u}.\,\frac{u^2}{2}\, du$$
$$= \frac{u^2 log\, u}{2} - \int \frac{u}{2}\, du$$
$$= \frac{u^2 log\, u}{2} = \frac{u^2}{4} + C$$

Question 5:

Obtain an integral of u log 2u.

Answer 5:

Suppose, I = $\int u\, log\, 2u\, du$

Integrating the equation by parts by taking log 2 u as first function and u as second function, we get,

$$I = log\, 2u \int u\, du - \int\left\{\left(\frac{d}{du}2log\, u\right)\int u\, du\right\}\, du$$
$$= log\, 2u.\,\frac{u^2}{2} - \int \frac{2}{2u}.\,\frac{u^2}{2}\, du$$
$$= \frac{u^2 log\, 2u}{2} - \int \frac{u}{2}\, du$$
$$= \frac{u^2 log\, 2u}{2} - \frac{u^2}{4} + C$$

Question 6:

Obtain an integral of u^2 log u.

Answer 6:

Suppose, I = $\int u^2\, log\, u\, du$

Integrating the equation by parts by taking log u as first function and u^2 as second function, we get,

$$I = log\, u \int u^2\, du - \int\left\{\left(\frac{d}{du}log\, u\right)\int u^2\, du\right\}\, du$$
$$= log\, u.\,\frac{u^3}{3} - \int \frac{1}{u}.\,\frac{u^2}{3}\, du$$
$$= \frac{u^3 log\, u}{3} - \int \frac{u^2}{3}\, du$$
$$= \frac{u^3 log\, u}{2} - \frac{u^3}{9} + C$$

Question 7:

Obtain an integral of u sin^{-1} u.

Answer 7:

Suppose, I = $\int u\, sin^{-1}\, u\, du$

Integrating the equation by parts by taking sin^{-1} u as first function and u as second function, we get,

$$I = sin^{-1}\, u \int u\, du - \int\left\{\left(\frac{d}{du}sin^{-1}\, u\right)\int u\, du\right\}\, du$$
$$= sin^{-1}\, u\frac{u^2}{2} - \int \frac{1}{\sqrt{1-u^2}}.\,\frac{u^2}{2}\, du$$
$$= \frac{u^2 sin^{-1}\, u}{2} + \frac{1}{2}\int \frac{-u^2}{\sqrt{1-u^2}}\, du$$
$$= \frac{u^2 sin^{-1}\, u}{2} + \frac{1}{2}\int\left\{\frac{1-u^2}{\sqrt{1-u^2}} - \frac{1}{\sqrt{1-u^2}}\right\}\, du$$
$$= \frac{u^2 sin^{-1}\, u}{2} + \frac{1}{2}\int\left\{\sqrt{1-u^2} - \frac{1}{\sqrt{1-u^2}}\right\}\, du$$
$$= \frac{u^2 sin^{-1}\, u}{2} + \frac{1}{2}\left\{\int\sqrt{1-u^2}\, du - \int\frac{1}{\sqrt{1-u^2}}\, du\right\}$$
$$= \frac{u^2 sin^{-1}\, u}{2} + \frac{1}{2}\left\{\frac{u}{2}\sqrt{1-u^2} + \frac{1}{2}sin^{-1}\, u - sin^{-1}\, u\right\} + C$$
$$= \frac{u^2 sin^{-1}\, u}{2} + \frac{u}{4}\sqrt{1-u^2} + \frac{1}{4}sin^{-1}\, u - \frac{1}{2}sin^{-1}\, u + C$$
$$= \frac{1}{4}(2u^2 - 1)sin^{-1}\, u + \frac{u}{4}\sqrt{1-u^2} + C$$

Question 8:

Obtain an integral of u tan^{-1} u

Answer 8:

Suppose, I = $\int u\, tan^{-1}\, u\, du$

Integrating the equation by parts by taking $\tan^{-1} u$ as first function and u as second function, we get,

$$I = tan^{-1}\, u \int u\, du - \int \left\{ \left(\frac{d}{du} tan^{-1}\, u \right) \int u\, du \right\}\, du$$
$$= tan^{-1}\, u\, \frac{u^2}{2} - \int \frac{1}{1+u^2} \cdot \frac{u^2}{2}\, du$$
$$= \frac{u^2 tan^{-1}\, u}{2} - \frac{1}{2} \int \frac{u^2}{1+u^2}\, du$$
$$= \frac{u^2 tan^{-1}\, u}{2} - \frac{1}{2} \int \left\{ \frac{u^2+1}{1+u^2} - \frac{1}{1+u^2} \right\}\, du$$
$$= \frac{u^2 tan^{-1}\, u}{2} - \frac{1}{2} \int \left\{ 1 - \frac{1}{1+u^2} \right\}\, du$$
$$= \frac{u^2 tan^{-1}\, u}{2} - \frac{1}{2} \int (u - tan^{-1}\, u) + C$$
$$= \frac{u^2 tan^{-1}\, u}{2} - \frac{u}{2} + \frac{1}{2}\, tan^{-1}\, u + C$$

Question 9:

Obtain an integral of u $\cos^{-1}$ u

Answer 9:

Suppose, I = $\int u\, cos^{-1}\, u\, du$

Integrating the equation by parts by taking $\cos^{-1}$ u as first function and u as second function, we get,

$$I = cos^{-1}\, u \int u\, du - \int \left\{ \left(\frac{d}{du} cos^{-1}\, u \right) \int u\, du \right\}\, du$$
$$= cos^{-1}\, u \frac{u^2}{2} - \int \frac{-1}{\sqrt{1-u^2}} \cdot \frac{u^2}{2}\, du$$
$$= \frac{u^2 cos^{-1}\, u}{2} - \frac{1}{2} \int \frac{1-u^2+1}{\sqrt{1-u^2}}\, du$$
$$= \frac{u^2 cos^{-1}\, u}{2} + \frac{1}{2} \int \left\{ \sqrt{1-u^2} + \frac{-1}{\sqrt{1-u^2}} \right\}\, du$$
$$= \frac{u^2 cos^{-1}\, u}{2} - \frac{1}{2} I_1 - \frac{1}{2}\, cos^{-1}\, u \ldots\ldots (1)$$

$$where\ I_1 = \int \sqrt{1-u^2}\, du$$
$$I_1 = u\, \sqrt{1-u^2} - \int \frac{d}{du}\, \sqrt{1-u^2} \int u\, du$$
$$I_1 = u\, \sqrt{1-u^2} - \int \frac{-2u}{2\sqrt{1-u^2}} . u\, du$$
$$I_1 = u\, \sqrt{1-u^2} - \int \frac{-u^2}{\sqrt{1-u^2}}\, du$$
$$I_1 = u\, \sqrt{1-u^2} - \int \frac{1-u^2-1}{\sqrt{1-u^2}}\, du$$
$$I_1 = u\, \sqrt{1-u^2} - \left\{ \int \sqrt{1-u^2}\, du + \int \frac{-du}{\sqrt{1-u^2}} \right\}$$
$$I_1 = u\, \sqrt{1-u^2} - \{ I_1 + cos^{-1}\, u \}$$
$$2I_1 = u\, \sqrt{1-u^2} - cos^{-1}\, u$$
$$I_1 = \frac{u}{2}\, \sqrt{1-u^2} - \frac{1}{2} cos^{-1}\, u$$

$Using\ in\ equation\ (1),\ we\ get,$

$$I = \frac{u^2 sin^{-1}\, u}{2} - \frac{1}{2} \left(\frac{u}{2}\, \sqrt{1-u^2} - \frac{1}{2}\, cos^{-1}\, u \right) - \frac{1}{2}\, cos^{-1}\, u$$
$$= \frac{(2u^2-1)}{4}\, cos^{-1}\, u - \frac{u}{4}\, \sqrt{1-u^2} + C$$

Question 10:

Obtain an integral of $(\sin^{-1} u)^2$

Answer 10:

Suppose, I = $\int (sin^{-1}\, u)^2 . 1\, du$

Integrating the equation by parts by taking $(\sin^{-1} u)^2$ as first function and 1 as second function, we get,

$$sin^{-1}\, u \int 1\, du - \int \left\{ \frac{d}{du} (sin^{-1}\, u)^2 . \int 1.du \right\}\, du$$
$$= (sin^{-1}\, u)^2 .\ u - \int \frac{2\,(sin^{-1}\, u)}{\sqrt{1-u^2}} .\ u\, du$$
$$= u . (sin^{-1}\, u)^2 + \int sin^{-1}\, u . \left(\frac{-2u}{\sqrt{1-u^2}} \right)\, du$$
$$= u . (sin^{-1}\, u)^2 + \left[sin^{-1}\, u \int \frac{-2u}{\sqrt{1-u^2}}\, du - \int \left\{ \left(\frac{d}{du} sin^{-1}\, u \right) \int \frac{-2u}{\sqrt{1-u^2}}\, du \right\}\, du \right]$$
$$= u . (sin^{-1}\, u)^2 + \left[sin^{-1}\, u . 2\sqrt{1-u^2} - \int \frac{1}{\sqrt{1-u^2}} . 2\sqrt{1-u^2}\, du \right]$$
$$= u . (sin^{-1}\, u)^2 + 2\sqrt{1-u^2}\, sin^{-1}\, u - \int 2\, du$$
$$= u . (sin^{-1}\, u)^2 + 2\sqrt{1-u^2}\, sin^{-1}\, u - 2u + C$$

Question 11:

Obtain an integral of $\frac{u\, cos^{-1}\, u}{\sqrt{1-u^2}}$

Answer 11:

Suppose, I = $\int \frac{u\, cos^{-1}\, u}{\sqrt{1-u^2}}\, du$

$$I = \frac{-1}{2} \int \frac{-2u}{\sqrt{1-u^2}} .\ cos^{-1} u\, du$$

Integrating the equation by parts by taking $\cos^{-1}$ u as first function and $\frac{-2u}{\sqrt{1-u^2}}$ as second function, we get,

$$I = \frac{-1}{2} \left[cos^{-1}\, u \int \frac{-2u}{\sqrt{1-u^2}}\, du - \int \left\{ \left(\frac{d}{du}\, cos^{-1}\, u \right) \int \frac{-2u}{\sqrt{1-u^2}}\, du \right\}\, du \right]$$
$$= \frac{-1}{2} \left[cos^{-1}\, u . 2\, \sqrt{1-u^2} - \int \frac{-1}{\sqrt{1-u^2}} . 2\, \sqrt{1-u^2}\, du \right]$$
$$= \frac{-1}{2} \left[2\, \sqrt{1-u^2}\, cos^{-1}\, u + \int 2\, du \right]$$
$$= \frac{-1}{2} \left[2\, \sqrt{1-u^2}\, cos^{-1}\, u + 2u \right] + C$$
$$= - \left[\sqrt{1-u^2}\, cos^{-1}\, u + u \right] + C$$

Question 12:

Obtain an integral of u $\sec^2$ u

Answer 12:

Suppose, I = $\int u\, sec^2\, u\, du$

Integrating the equation by parts by taking u as first function and $\sec^2$ u as second function, we get,

$$u \int sec^2\, u\, du - \int \left\{ \left(\frac{d}{du} u \right) \int sec^2\, u\, du \right\}\, du$$

$u \int sec^2 u\, du - \int \{ \{ \frac{d}{du}. u \} \int sec^2 u\, du \}\, du$
$= u\, tan\, u - \int 1.\, tan\, u\, du$
$= u\, tan\, u - log\, |cos\, u| + C$

Question 13:

Obtain an integral of tan $^{-1}$ u

Answer 13:

Suppose, I = $\int tan^{-1} u\, du$

Integrating the equation by parts by taking tan $^{-1}$ u as first function and 1 as second function, we get,

$tan^{-1} u \int 1\, du - \int \{ (\frac{d}{du}\, tan^{-1} u) \int 1\, du \}\, du$
$= tan^{-1} u.\ u - \int \frac{1}{1+u^2}.\ u\, du$
$= tan^{-1} u.\ u - \frac{1}{2} \int \frac{2u}{1+u^2}.\ du$
$= u\, tan^{-1} u - \frac{1}{2}\, log\, |1+u^2| + C$
$= u\, tan^{-1} u - \frac{1}{2}\, log\, (1+u^2) + C$

Question 14:

Obtain an integral of u (log u) ^{2}.

Answer 14:

Suppose, I =

Integrating the equation by parts by taking (log u) ^{2} as first function and 1 as second function, we get,

$I = (log\, u)^2 \int u\, du - \int [\{ (\frac{d}{du}\, log\, u)^2 \} \int u\, du]\, du$
$= \frac{u^2}{2} (log\, u)^2 - \left[\int 2 log\, u.\ \frac{1}{u}.\ \frac{u^2}{2}\, du \right]$
$= \frac{u^2}{2} (log\, u)^2 - \int u\, log\, u\, du$

Integrating the equation again by parts, we get,

$I = \frac{u^2}{2} (log\, u)^2 \int u\, du - [log\, u \int u\, du - \{ (\frac{d}{du}\, log\, u) \int u\, du \}\, du]$
$= \frac{u^2}{2} (log\, u)^2 - \left[\frac{u^2}{2} - log\, u - \int \frac{1}{u}.\ \frac{u^2}{2}\, du \right]$
$= \frac{u^2}{2} (log\, u)^2 - \frac{u^2}{2} (log\, u) + \frac{1}{2} \int u\, du$
$= \frac{u^2}{2} (log\, u)^2 - \frac{u^2}{2} (log\, u) + \frac{u^2}{4} + C$

Question 15:

Obtain an integral of (u ^{2} + 1) log u

Answer 15:

Suppose, I = $\int (u^2+1) log\, u\, du = \int u^2\, log\, u\, du + \int log\, u\, du$

$Suppose,\ I = I_1 + I_2 + \ldots\ldots (1)$
$Where,\ I_1 = \int u^2\, log\, u\, du\ and\ I_2 = \int log\, u\, du$
$I_1 = \int u^2\, log\, u\, du$

Integrating the equation by parts by taking u as first function and u ^{2} as second function, we get,

$I_1 = (log\, u) - \int u^2\, du - \int \{ (\frac{d}{du} log\, u) \int u^2\, du \}\, du$
$= log\, u.\ \frac{u^3}{3} - \int \frac{1}{u}.\ \frac{u^3}{3}\, du$
$= \frac{u^3}{3}\, log\, u - \frac{1}{3} (\int u^2\, du)\, du$
$= \frac{u^3}{3}\, log\, u - \frac{u^3}{9} + C_1 \ldots . (2)$
$I_2 = \int log\, u\, du$

Integrating the equation by parts by taking u as first function and u ^{2} as second function, we get,

$I_2 = (log\, u) - \int 1\, du - \int \{ (\frac{d}{du} log\, u) \int 1\, du \}$
$= log\, u. u - \int \frac{1}{u}. u\, du$
$= u\, log\, u - \int 1\, du$
$= u\, log\, u - u + C_2 \ldots . (3)$

Substituting equations (2) and (3) in equation (1), we get,

$I = \frac{u^3}{3}\, log\, u - \frac{u^3}{9} + C_1 + u\, log\, u - u + C_2$
$= \frac{u^3}{3}\, log\, u - \frac{u^3}{9} + u\, log\, u - u + (C_1 + C_2)$
$= \left(\frac{u^3}{3} + u \right) log\, u - \frac{u^3}{9} - C$

Question 16:

Obtain an integral of e ^{u} (sin u + cos u)

Answer 16:

Suppose, I = $\int e^u(sin\ u+cos\ u)\ du$

$$Suppose,\ f(u)=sin\ u$$
$$f'(u)=cos\ u$$
$$I=\int e^u\{f(u)+f'(u)\}\ du$$
$$As\ we\ know,$$
$$\int e^u\{f(u)+f'(u)\}\ du=e^uf(u)+C$$
$$I=e^u\ sin\ u+C$$

Question 17:

Obtain an integral of $\frac{e^u}{(1+u)^2}$

Answer 17:

Suppose, I = $\int\frac{u\ e^u}{(1+u)^2}\ du=\int e^u\left\{\frac{u}{(1+u)^2}\right\}\ du$

$$=\int e^u\left\{\frac{1+u-1}{(1+u)^2}\right\}\ du$$
$$=\int e^u\left\{\frac{1}{1+u}-\frac{1}{(1+u)^2}\right\}\ du$$
$$Suppose,\ f(u)=\frac{1}{1+u},\quad f'(u)=\frac{-1}{(1+u)^2}$$
$$\int\frac{u\ e^u}{(1+u)^2}\ du=\int e^u\{f(u)+f'(u)\}\ du$$
$$As\ we\ know,$$
$$\int e^u\{f(u)+f'(u)\}\ du=e^u\ f(u)+C$$
$$\int\frac{u\ e^u}{(1+u)^2}\ du=\frac{e^u}{1+u}+C$$

Question 18:

Obtain an integral of $e^u\left(\frac{1+sin\ u}{1+cos\ u}\right)$

Answer 18:

$$e^u\left(\frac{1+sin\ u}{1+cos\ u}\right)$$
$$=e^u\left(\frac{sin^2\ \frac{u}{2}+cos^2\ \frac{u}{2}+2sin\ \frac{u}{2}\ cos\ \frac{u}{2}}{2\ cos^2\ \frac{u}{2}}\right)$$
$$=\frac{e^u\left(sin\ \frac{u}{2}+cos\ \frac{u}{2}\right)^2}{2cos^2\ \frac{u}{2}}$$
$$=\frac{1}{2}e^u\left(\frac{sin\ \frac{u}{2}+cos\ \frac{u}{2}}{cos\ \frac{u}{2}}\right)^2$$
$$=\frac{1}{2}e^u\left[tan\ \frac{u}{2}+1\right]^2$$
$$=\frac{1}{2}e^u\left[1+tan\ \frac{u}{2}\right]^2$$
$$=\frac{1}{2}e^u\left[1+tan^2\ \frac{u}{2}+2tan\ \frac{u}{2}\right]$$
$$=\frac{1}{2}e^u\left[sec^2\ \frac{u}{2}+2tan\ \frac{u}{2}\right]$$
$$\frac{e^u(1+sin\ u)\ du}{(1+cos\ u)}=\left[\frac{1}{2}sec^2\ \frac{u}{2}+2tan\ \frac{u}{2}\right]\quad .....(1)$$
$$Suppose,\ tan\ \frac{u}{2}=f(u)\ so\ f'(u)=\frac{1}{2}sec^2\ \frac{u}{2}$$

As we know,

$$\int e^u\{f(u)+f'(u)\}\ du=e^u\ f(u)+C$$

Considering equation (1), we get,

$$\int\frac{e^u(1+sin\ u)\ du}{(1+cos\ u)}=e^utan\ \frac{u}{2}+C$$

Question 19:

Obtain an integral of $e^u\left(\frac{1}{u}-\frac{1}{u^2}\right)$

Answer 19:

Suppose, I = $\int e^u\left(\frac{1}{u}-\frac{1}{u^2}\right)\ du$

$$Suppose,\ \frac{1}{u}=f(u)\quad f'(u)=\frac{-1}{u^2}$$
$$As\ we\ know,$$
$$\int e^u\{f(u)+f'(u)\}\ du=e^u\ f(u)+C$$
$$I=\frac{e^u}{u}+C$$

Question 20:

Obtain an integral of $\frac{(u-3)e^u}{(u-1)^3}$

Answer 20:

$$\int e^u \left\{ \frac{(u-3)}{(u-1)^3} \right\} du = \int e^u \left\{ \frac{(u-3)}{(u-1-2)^3} \right\} du$$

$$= \int e^u \left\{ \frac{1}{(u-1)^2 - \frac{2}{(u-1)^3}} \right\} du$$

$$f(u) = \frac{1}{(u-1)^2} \quad f'(u) = \frac{-2}{(u-1)^3}$$

As we know,

$$\int e^u \{f(u) + f'(u)\} \ du = e^u \ f(u) + C$$

$$\int e^u \left\{ \frac{(u-3)}{(u-1)^3} \right\} du = \frac{e^u}{(u-1)^2} + C$$