Q.1) 5×10^{-8} C and -3×10^{-8} C are the two charges located 16 cm apart from each other. At what point(s) between these two charges the electric potential is zero?

Soln.: Given,
$q_1 = 5 \times 10^{-8}$ C
$q_2 = -3 \times 10^{-8}$ C

The two charges are at a distance, d = 16cm = 0.16m from each other.

As shown in the figure, let us consider a point P over the line joining charges q_1 and q_2.

Let, distance of the considered point P from q_1 be 'r'

Let, point P has zero electric potential (V).

The electric potential at point P is the summation of potentials due to charges q_1 and q_2.

Therefore, $V = \frac{1}{4\pi\epsilon_o}.\frac{q_1}{r} + \frac{1}{4\pi\epsilon_o}.\frac{q_2}{d-r}$ ....(1)

Here,

ϵ_o = permittivity of free space.

Putting V = 0, in eqn. (1), we get,

$0 = \frac{1}{4\pi\epsilon_o}.\frac{q_1}{r} + \frac{1}{4\pi\epsilon_o}.\frac{q_2}{d-r}$

$\frac{1}{4\pi\epsilon_o}.\frac{q_1}{r} = -\frac{1}{4\pi\epsilon_o}.\frac{q_2}{d-r}\ \frac{q_1}{r} = -\frac{q_2}{d-r}\ \frac{5\times10^{-8}}{r} = -\frac{(-3\times10^{-8})}{0.16-r}$

5(0.16 – r) = 3r

0.8 = 8r

r = 0.1m = 10 cm.

Therefore, at a distance of 10 cm from the positive charge the potential is zero between the two charges.

Let us assume a point P at a distance 's' from the negative charge be outside the system, having potential zero.

So, for the above condition, potential is given by –

$V = \frac{1}{4\pi\epsilon_o}.\frac{q_1}{s} + \frac{1}{4\pi\epsilon_o}.\frac{q_2}{s-d}$ ....(2)

Here,

ϵ_o = permittivity of free space.

For V = 0, eqn. (2) can be written as :

$0 = \frac{1}{4\pi\epsilon_o}.\frac{q_1}{s} + \frac{1}{4\pi\epsilon_o}.\frac{q_2}{s-d}$

$\frac{1}{4\pi\epsilon_o}.\frac{q_1}{s} = -\frac{1}{4\pi\epsilon_o}.\frac{q_2}{s-d}\ \frac{q_1}{s} = -\frac{q_2}{s-d}\ \frac{5\times10^{-8}}{s} = -\frac{(-3\times10^{-8})}{s-0.16}$

5(s – 0.16) = 3s

0.8 = 2s

S = 0.4 m = 40 cm.

Therefore, at a distance of 40 cm from the positive charge outside the system of charges, the potential is zero.

Q.2) A regular hexagon having sides equal to 10 cm, contains charges of 5 μC at its each vertices.

Find the potential at the centre of this regular hexagon.

Soln.:

The figure shows, regular hexagon containing charges q, at each of its vertices.

Here,

q = 5 μC = 5×10^{-6} C.

Length of each side of hexagon, AB =BC = CD = DE = EF = FA = 10 cm.

Distance of the vertices from the centre O, d = 10 cm.

Electric potential at point O,

V = $\frac{1}{4\pi\epsilon_o}.\frac{6xq}{d}$

Here,

ϵ_o = Permittivity of free space and $\frac{1}{4\pi\epsilon_o}$ = 9 x 10^9 Nm^2C^{-2}

V = $\frac{9x10^9x6x5x10_{-6}}{0.1}$ = 2.7 x 10^6 V.

Q.3) Charges 2 μC and -2 μC at a point A and B, respectively are 6 cm apart.

(1) Obtain an equipotential surface of the system.

(2) At every point on the surface find the direction of electric field.

Soln.:

(1) An equipotential surface is defined as the surface over which the total potential is zero. In the given question this plane is normal to line AB. The plane is located at the mid – point of the line AB as the magnitude of the charges are same.

(2) At every point on this surface the direction of the electric field is normal to the plane in the direction of AB.

Q.4) A conductor is in the shape of a sphere with radius 12 cm having charge 1.6 x 10^{-7}C uniformly distributed over its surface. What will be the electric field ?

(1) Inside the sphere.

(2) Just outside the sphere.

(3) At a point 18 cm from the centre of sphere.

Soln.:

(1) Given,

Radius of spherical conductor, r = 12cm = 0.12m

Charge is distributed uniformly over the surface, q = 1.6 x 10^{-7} C.

Electric field inside a spherical conductor is zero.

(2) Electric field E, just outside the conductor is given by the relation,

E = $\frac{1}{4\pi\epsilon_o}.\frac{q}{r^2}$

Here,

= permittivity of free space and $\frac{1}{4\pi\epsilon_o}$ = 9 x 10^9 Nm^2C^{-2}

Therefore,

E = $\frac{9\times10^9\times1.6\times10^{-7}}{(0.12)^2}$

Therefore, just outside the sphere the electric field is 4.4 x 10^4 NC^{-1}.

(3) From the centre of sphere the electric field at a point 18m = E_1.

From the centre of sphere the distance of point d = 18 cm = 0.18m.

E_1 = $\frac{1}{4\pi\epsilon_o}.\frac{q}{d^2}$ = $\frac{9\times10^9\times1.6\times10^{-7}}{((1.8\times10)^{-2})^2}$ = 4.4 x 10^4 NC^{-1}

So, from the centre of sphere the electric field at a point 18 cm away is 4.4 x 10^4 NC^{-1}.

Q.5) What will be the capacitance of a parallel plate capacitor with air between the plates having capacitance of 8pF (1pF = 10^{-12} F). How much the capacitance of the capacitor will change if the distance between the plate is reduced to half, and the space between the plates is filled with a substance of dielectric constant 6?

Soln.: Given,

Capacitance, C = 8pF.

In first case the parallel plates are at a distance ‘d’ and is filled with air.

Air has dielectric constant, k = 1

Capacitance, C = $\frac{k\times\epsilon_o\times A}{d}$ = $\frac{\epsilon_o\times A}{d}$ ... eq(1)

Here

Here,

A = area of each plate

ϵ_o = permittivity of free space.

Now, if the distance between the parallel plates is reduced to half, then $d_1 = d/2$

Given, dielectric constant of the substance, $k_1 = 6$

Hence, the capacitance of the capacitor,

$C_1 = \frac{k_1 \times \epsilon_o \times A}{d_1} = \frac{6\epsilon_o \times A}{d/2} = \frac{12\epsilon_o A}{d}$ ... (2)

Taking ratios of eqns. (1) and (2), we get,

C_1 = 2 x 6 C = 12 C = 12 x 8 pF = 96pF.

Hence, capacitance between the plates is 96pF.

Q.6) Three capacitors connected in series have capacitance of 9pF each.

(1) What can be total capacitance of this combination?

(2) Find the potential difference across each capacitor if they are connected to a 120v supply?

Soln.:

(1) Given,

Capacitance of the three capacitors, C = 9 pF

Equivalent capacitance (c_{eq}) is the capacitance of the combination of the capacitors given by

$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C} = \frac{3}{9} = \frac{1}{3}$

$\frac{1}{C_{eq}} = \frac{1}{3}$ = C_{eq} = 3 pF

Therefore, total capacitance = 3pF.

(2) Given, supply voltage, V = 100v

The potential difference (V_1) across the capacitors will be equal to one – third of the supply voltage.

Therefore, $V_1 = \frac{V}{3} = \frac{120}{3}$ = 40V.

Hence, the potential difference across each capacitor is 40V.

Q.7) In parallel combination three capacitors of capacitances 2pF, 3pF and 4pF are connected.

(1) What will be the total capacitance of this combination?

(2) If the connection is connected to 100V supply, find the charge on each capacitor.

Soln.:

(1) Given, C_1 = 2pF, C_2 = 3pF and C_3 = 4pF.

Equivalent capacitance for the parallel combination is given by C_{eq} .

Therefore, $C_{eq} = C_1 + C_2 + C_3$ = 2 + 3 + 4 = 9pF

Hence, total capacitance of the combination is 9pF.

(2) supply voltage, V = 100v

The three capacitors are having the same voltage, V = 100v

q = vc

where,

q = charge

c = capacitance of the capacitor

v = potential difference

for capacitance, c = 2pF

q = 100 x 2 = 200pC = 2×10^{-10}C for capacitance, c = 3pF

q = 100 x 3 = 300pC = 3×10^{-10}C

for capacitance, c = 4pF

q = 100 x 4 = 400pC = 4×10^{-10} C

Q.8) A parallel plate capacitor having air in between the plates each having area of 6×10^{-3} m ^{2} and distance between them is 3 mm. Find the capacitance of the capacitor. What will be the charge on each plate if the capacitor is connected to a 100v source?

Soln.: Given,

The area of plate of the capacitor, A = 6×10^{-3} m^2

Distances between the plates, d = 3mm = 3×10^{-3} m

Voltage supplied, V = 100v

Capacitance of a parallel plate capacitor is given by, C = $\frac{\epsilon \times A}{d}$

Here,

ε = permittivity of free space = 8.854×10^{-12} N^{-1} m^{-2} C^{-2}

C = $\frac{8.854 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$ = 17.81×10^{-12} F = 17.71 pF.

Therefore, each plate of the capacitor is having a charge of

q = VC = 100 × 17.81×10^{-12} C = 1.771×10^{-9} C

Q 9 :Considering the same capacitor as in Q 8, when a thick mica sheet, 3mm wide is placed between the plates (Dielectric constant = 6)

(a)With constant voltage supply

(b) After disconnecting the voltage supply

Answer 2.9:

(a) Dielectric constant of the mica sheet, k = 6

If voltage supply remained connected, voltage between two plates will be

constant.

Supply voltage, V = 100 V

Initial capacitance, C = 1.771×10^{-11} F

New capacitance, C_1 = kC = 6 × 1.771×10^{-11} F = 106 pF

New charge, q_1 = C_1V = 106 × 100 pC = 1.06×10^{-8} C

Potential across the plates remains 100 V.

(b) Dielectric constant, k = 6

Initial capacitance, C = 1.771×10^{-11} F

New capacitance, C_1 = kC = 6 × 1.771×10^{-11} F = 106 pF

If supply voltage is removed, then there will be constant amount of charge

in the plates.

Charge = 1.771×10^{-9} C

Potential across the plates is given by,

V1 = q/C1 = $\frac{1.771 \times 10^{-9}}{106 \times 10^{-12}}$

= 16.7 V

Q.10) How much of electrostatic energy is stored in the capacitor if the capacitor with 12pF of capacitance is connected to a 50v battery?

Soln.: Given,

Capacitance of the capacitor, C = 12pF = 12×10^{-12} F

Potential difference, V = 50 V

Electrostatic energy stored in the capacitor is given by the relation,

$E = \frac{1}{2}CV^2 = \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2 \text{ J} = 1.5 \times 10^{-8} \text{ J}$

Therefore, the electrostatic energy stored in the capacitor is 1.5×10^{-8} J.

was disconnected.

Q.11) 200V of source is connected to a 600pF capacitor. Later, the source is disconnected to the 600pf capacitor and connected to another 600pF capacitor. How much of electrostatic energy is lost in the process?

Soln.: Given,

Capacitance, C = 600pF

Potential difference, V = 200v

Electrostatic energy stored in the capacitor is given by :

$E_1 = \frac{1}{2}CV^2 = \frac{1}{2} \times (600 \times 10^{-12}) \times (200)^2 \text{J} = 1.2 \times 10^{-5} \text{ J}$

Acc. to the question, the source is disconnected to the 600pF and connected to another capacitor of 600pF, then equivalent capacitance (C_{eq}) of the combination is given by,

$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C}$

$\frac{1}{C_{eq}} = \frac{1}{600} + \frac{1}{600}$

$= \frac{2}{600} = \frac{1}{300}$

$C_{eq} = 300\text{pF}$

New electrostatic energy can be calculated by:

$E_2 = \frac{1}{2}CV^2 = \frac{1}{2} \times 300 \times (200)^2 \text{ J} = 0.6 \times 10^{-5} \text{ J}$

Loss in electrostatic energy,

$E = E_1 - E_2$

$E = 1.2 \times 10^{-5} - 0.6 \times 10^{-5} \text{ J} = 0.6 \times 10^{-5} \text{ J} = 6 \times 10^{-6} \text{ J}$

Therefore, the electrostatic energy lost in the process is 6×10^{-6} J.