Q.1. (i) 4^{-3}

(ii) $(-5)^{-5}$

(iii) $\left(\frac{1}{2}\right)^{-4}$

Solution:

(i) 4^{-2}

$(4)^{-2} = \frac{1}{4^2} = \frac{1}{16}$

(ii) $(-5)^{-2}$

$(-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{25}$

(iii) $\left(\frac{1}{2}\right)^{-4}$

$\left(\frac{1}{2}\right)^{-2} = \left(\frac{2}{1}\right)^2 = (2)^2 = 4$

Formula: $a^{-m} = \frac{1}{a^m}$

Q.2. (i) $(-5)^4 \div (-5)^7$

(ii) $\left(\frac{1}{4^3}\right)^2$

(iii) $(-4)^3 \times \left(\frac{5}{4}\right)^3$

(iv) $(2^{-7} \div 2^{-10}) \times 2^{-5}$

(v) $3^{-3} \times (-7)^{-3}$

Solution:

(i) $(-5)^4 \div (-5)^7$

$(-5)^4 \div (-5)^7 = (-5)^{4-7}$
$= (-5)^{-3}$
$= \frac{1}{(-5)^3}$

(ii) $\left(\frac{1}{4^3}\right)^2$

$\left(\frac{1}{4^3}\right)^2 = \frac{1^2}{(4^3)^2}$
$= \frac{1}{4^{3\times2}}$
$= \frac{1}{4^6}$

(iii) $(-4)^3 \times \left(\frac{5}{4}\right)^3$

$(-4)^2 \times \left(\frac{5}{4}\right)^2 = (-4)^2 \times \frac{5^2}{4^2}$

$= [(-1)^2 \times 4^2] \times \frac{5^2}{4^2}$

$= 4^{2-2} \times 5^2 = 4^0 \times 5^2 = 5^2$

(iv) $(2^{-7} \div 2^{-10}) \times 2^{-5}$

$(2^{-7} \div 2^{-10}) \times 2^{-5} = 2^{-7-(-10)} \times 2^{-5}$

$= 2^{-7+10} \times 2^{-5}$

$= 2^3 \times 2^{-5} = 2^{3-5}$

$= 2^{-2} = \frac{1}{2^2}$

(v) $3^{-3} \times (-7)^{-3}$

$3^{-3} \times (-7)^{-3} = \frac{1}{3^3} \times \frac{1}{(-7)^3}$

$= \frac{1}{[3 \times (-7)]^3} = \frac{1}{(-21)^3}$

Formulae:

1. $(a^m)^n = a^{m \times n}$

2. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

3. $(ab)^m = a^m b^m$

4. $a^m \div a^n = a^{m-n}$

5. $a^m \times a^n = a^{m+n}$

Q.3. Find the value of:

(i) $(3^0 + 4^{-1}) \times 2^2$

(ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$

(v) $\left[\left(\frac{-2}{3}\right)^{-2}\right]^2$

Solution:

(i) $(3^0 + 4^{-1}) \times 2^2$

$(1 + \frac{1}{4}) \times 2^2$

$= \left(\frac{4+1}{4}\right) \times 2^2 = \frac{5}{4} \times 2^2$

$= 5 \times 2^{2-2}$

$= 5 \times 2^0 = 5 \times 1 = 5$

(ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

$(2^{-1} \times 4^{-1}) \div 2^{-2} = (\frac{1}{2^1} \times \frac{1}{4^1}) \div 2^{-2}$

$= (\frac{1}{2} \times \frac{1}{2^2}) \div 2^{-2} = \frac{1}{2^3} \div 2^{-2}$

$= 2^{-3} \div 2^{-2} = 2^{-3-(-2)} = 2^{-3+2} = 2^{-1}$

$= \frac{1}{2}$

(iii) $(\frac{1}{2})^{-2} + (\frac{1}{3})^{-2} + (\frac{1}{4})^{-2}$

$(\frac{1}{2})^{-2} + (\frac{1}{3})^{-2} + (\frac{1}{4})^{-2} = (2^{-1})^2 + (3^{-1})^2 + (4^{-1})^2$

$= 2^{-1 \times (-2)} + 3^{-1 \times (-2)} + 4^{-1 \times (-2)}$

$= 2^2 + 3^2 + 4^2 = 4 + 9 + 16 = 29$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$

$(3^{-1} + 4^{-1} + 5^{-1})^0 = (\frac{1}{3} + \frac{1}{4} + \frac{1}{5})^0$

$= (\frac{20+15+12}{60})^0 = (\frac{47}{60})^0 = 1$

(v) $[(\frac{-2}{3})^{-2}]^2$

$[(\frac{-2}{3})^{-2}]^2 = (\frac{-2}{3})^{-2 \times 2}$

$= (\frac{-2}{3})^{-4} = (\frac{-3}{2})^4 = \frac{81}{16}$

Q.4. Evaluate:

(i) $\frac{8^{-1} \times 5^3}{2^{-4}}$

(ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

Solution:

(i) $\frac{8^{-1} \times 5^3}{2^{-4}}$

$\frac{8^{-1} \times 5^3}{2^{-4}} = \frac{(2^3)^{-1} \times 5^3}{2^{-4}} = \frac{2^{-3} \times 5^3}{2^{-4}}$

$= 2^{-3(-4)} \times 5^3 = 2^{-3+4} \times 5^3$

$= 2 \times 125 = 250$

(ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

$(5^{-1} \times 2^{-1}) \times 6^{-1} = (\frac{1}{5} \times \frac{1}{2}) \times \frac{1}{6}$

$= \frac{1}{10} \times \frac{1}{6} = \frac{1}{60}$

Q.5. Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Solution:

$5^m \div 5^{-3} = 5^5$ $5^m \div 5^{-3} = 5^5$
$\Rightarrow 5^{m-(-3)} = 5^5$ $\Rightarrow 5^{m-(-3)} = 5^5$
$\Rightarrow 5^{m+3} = 5^5$ $\Rightarrow 5^{m+3} = 5^5$
$\Rightarrow m + 3 = 5$
$\Rightarrow m = 5 - 3$
$\Rightarrow m = 2$

Q.6. Evaluate:

(i) $\left[\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right]^{-1}$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

Solution:

(i) $\left[\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right]^{-1}$

$\left[\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right]^{-1} = \left[\left(\frac{3}{1}\right)^1 - \left(\frac{4}{1}\right)^1\right]$ ($because\, a^{-m} = \frac{1}{m}$)
$= [3 - 4] = -1$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

$\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}}$
$= 5^{-7-(-4)} \times 8^{-4-(-7)}$
$= 5^{-7+4} \times 8^{-4+7} = 5^{-3} \times 8^3 = \frac{8^3}{5^3}$
$= \frac{512}{125}$

Q.7. Simplify:

(i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $(t \neq 0)$

(ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Solution:

(i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $(t \neq 0)$

$\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} = \frac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}}$
$= \frac{5^{2-(-3)-1} \times t^{-4-(-8)}}{2}$
$= \frac{5^{2+3-1} \times t^{-4+8}}{2} = \frac{5^4 \times t^4}{2}$
$= \frac{625}{2} t^4$

(ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} = \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}}$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5+3}}{5^{-7} \times 2^{-5} \times 3^{-5}} = \frac{3^{-5} \times 2^{-5} \times 5^{-2}}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

$$= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-2-(-7)}$$

$$= 3^{-5+5} \times 2^{-5+5} \times 5^{-2+7} = 3^0 \times 2^0 \times 5^5 = 1 \times 1 \times 3125$$

$$= 3125$$